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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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Block 20 Continued:

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AN EVALUATION OF DIRECT CURRENT ELECTROMAGNETIC
PROPULSION IN SEAWATER

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ABSTRACT

Electromagnetic seawater thrusters may be classified in one of
three general categories: internal duct dc; external field dc, and
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maneuvering small ($\sim$10 ton displacement) submersibles.

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NOMENCLATURE

K_s = $C_D A_s \rho/2$ = constant of proportionality: $D = K_s v_s^2$ $(Nt/(m/s)^2)$

v_s = relative velocity between submerged body and water (m/s)

v_{out} = discharge velocity (m/s)

v_p = velocity in pumping region of duct (m/s)

C_D = drag coefficient (dimensionless)

v_p = velocity in pumping region of duct (m/s)

A_s = effective drag cross-sectional area (m^2)

ρ = density of water (kg/m^3)

σ = electrical conductivity of water (mho/m)

D = drag (newtons)

T = thrust (newtons)

J = duct electrical current density (amp/m^2)

I = duct electrical current (amps)

V = duct voltage (volts)

L = electrode length (meters)

B = duct magnetic field (tesla)

b = one-half electrode spacing (m)

a = one-half duct width along B-field direction (m)

α = aspect ratio = b/a (dimensionless)

A_p = 4ab = cross-sectional area of duct in pumping region (m^2)

A_{out} = cross-sectional area of discharge (m^2)

A_c = cross-sectional area of inlet cone

$\dot{m}$ = mass flow rate thru duct (kg/s)

η = Tv_s/IV = efficiency (dimensionless)

k_1 = derived parameter = v_p/v_s (dimensionless)

P_o = free-stream pressure (Nt/m^2)

P_z = nozzle pressure (Nt/m^2)

P_{in} = pressure at pump inlet (Nt/m^2)

ΔP_z = $P_z - P_o$ = pressure drop through nozzle (Nt/m^2)

ΔP_{in} = $P_o - P_{in}$ = inlet pressure drop (Nt/m^2)

ΔP_p = $P_z - P_{in}$ = pump head (Nt/m^2)

f = factor appearing in voltage, current and efficiency expressions (dimensionless)

N = interaction parameter (dimensionless)

W_f = fluid power (watts)

η_p = propulsive efficiency (dimensionless)

η_e = electrical efficiency (dimensionless)

ε = v_p/v_s (dimensionless)

1.0 INTRODUCTION

This study was undertaken to investigate the practicality of internal duct, direct current electromagnetic propulsion of submersible ocean vessels, ranging from full size submarines down to small maneuverable underwater platforms. The scope of this study is limited to a broad overview of performance characteristics rather than detailed design considerations: the emphasis here is on rapid assessment of trade-offs between major design parameters, such as magnetic field intensity, electrode length and duct size. Performance and efficiency calculations do not, therefore, include minor hydrodynamic and magnetohydrodynamic (MHD) effects, and the resulting curves should be used for general comparisons and trade-off analyses. These curves, for instance, enable the reader to quickly determine overall thrust efficiency as a function of duct area, magnetic field strength, electrode length, and multiple ducting. This information is presented for each of three submersible classes: full, one-third, and one-tenth sizes corresponding to hull diameters of about 10, 3, and 1 meters, respectively.* The assumed drag characteristics for these three sizes are given in Fig. 1.

As will be shown later, overall performance is a strong function of the magnetic field intensity, and the magnitude of field used for the performance curves given here ranges from conventional (0.5 T) to super-conducting (5 T) excitation sources. The impact of conductivity enhancement via seeding with hydrochloric acid (HCℓ) is also presented along with estimates of the volume of evolved gases released by electrolysis at the electrode surfaces. Finally, duct dimensions are restricted to reasonable ranges of engineering capability, limited by the volume requirements for the magnetic field. It does not seem reasonable, for instance, to consider a thruster design that requires 10 Tesla (super-

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*See Appendix A-1 for a discussion of hull size, shape, displacement, and drag.

conducting excitation) throughout a duct interior of several hundred
cubic meters. Duct dimensions, therefore, range from two to ten meters
long and up to one square meter of cross-sectional area.

2.0 CONCLUSIONS AND RECOMMENDATIONS

Electromagnetic thrusters offer a silent, nearly undetectable means of submersible propulsion. Based upon efficiency calculations included in this study though, it seems unlikely that a practical thruster system could be developed for a full size (2000 ton, 10 m hull diameter) submarine. The chief reason for this conclusion lies in the difficulty of establishing high magnetic field intensities throughout the active pumping region contained between the electrodes. A reasonably efficient thruster cannot be designed and built until a superconducting dewar/winding capable of functioning reliably in a submersible environment has been demonstrated. Once this has been achieved, then efficiencies of ten percent or more may be possible for reduced hull sizes of one to three meters diameter, provided that speed is limited to approximately ten knots or less. It may be feasible to use electromagnetic propulsion for manuevering small submersible platforms requiring either modest continuous thrust or occasional bursts of high thrust levels.

In general, efficiency may be improved by:

- minimizing thrust requirements.....small hulls operating at low velocities.

- using large areas, low velocity thrusters.....multiple thrust-ducts are the most practical way of achieving this objective without requiring excessive cross-sectional area per duct.

- operating at high magnetic fields.....preferably at field intensities much greater than that of conventional iron-copper magnetic circuits--hence, superconducting excitation is mandated.

- enhancing conductivity.....seeding seawater with a strong electrolyte such as hydrochloric acid has a pronounced

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effect upon overall efficiency. The rate and duration of seeding is limited, of course, by on-board storage capacity.

Development of an electromagnetic thruster should proceed in several stages:

- Define the mission requirements -- range, speed, depth, hull drag, power source, and acceptable levels of magnetic leakage.

- Demonst e the capability of establishing a high-intensity ($\sim$5 Tesla) magnetic field throughout a significant duct volume. This requires design and fabrication of a superconducting dewar/winding capable of functioning reliably in a submersible environment. One possible annular thruster configuration is shown in Figure 33 where a superconducting toroidal* field winding establishes high fields in the pump annulus with relatively low leakage into the surrounding region.

- Construct and test a prototype thruster.

- For military applications, the possibility of detection should receive critical evaluation, and provision should be made to evaluate magnetic field leakage, hydrolysis, and chemical activity (such as pH shift due to seeding) as potential means of detection.

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*Large superconducting toroidal magnets have, in fact, been successfully used in radiotherapy.[9]

3.0 BACKGROUND

3.1 General Description

Electromagnetic (EM) propulsion of sea vessels may be divided into three categories: internal duct dc; external field dc; and peristaltic ac induction. All three types of propulsion have been analyzed and discussed in the literature.[1-4] Here we present a brief description of these three.

The first type of thruster--internal duct dc--consists of electrodes mounted within a propulsion duct such that current (in the conducting fluid) established between the electrodes is perpendicular to a magnetic field established within the same region, but perpendicular to the direction of current flow. (See Fig. 2 for a pictorial diagram.) Interaction of current and magnetic field produces a mutually perpendicular pressure gradient or force directed along the duct axis. If the duct is open at either end, this gradient causes fluid flow and, hence, an axial reaction thrust. While in principle electromagnetic propulsion may be achieved quite easily by passing current between two electrodes located within a magnetic field, in practice the low value of electrical conductivity of seawater places a severe constraint on overall thrust efficiency that may be achieved.

The second type of thruster--external dc field--does not utilize thrust ducts. Instead, exterior electrodes are mounted along the hull and an external magnetic field is established such that interaction of electrode current and field produces external pressure gradients along the submersible's centerline, thereby creating propulsive thrust. Once again, though, the low conductivity of seawater limits ultimate thrust efficiency. A useful system requires superconducting excitation that establishes high magnetic fields throughout large volumes of water:

however, not only are such superconducting magnets expensive and difficult to build, but the resulting unshielded, easily detectible fields are most likely not suitable for military applications.

Finally, the third type of electromagnetic thruster--peristaltic ac induction--utilizes a flexible membrane to separate two fluid regions: one, an annular chamber containing a highly conducting fluid such as liquid metal; the other an inner cylindrical chamber open to the surrounding water. Radial ac magnetic fields induce circumferential ac currents in the outer (highly-conducting) annular chamber and these currents interacting with the induction fields generate pressure waves that impart axial motion to the conducting liquid. This, in turn, distorts the flexible membrane, thereby imparting axial motion to the water core. In short, traveling waves along the membrane squeeze the inner core, squirting water axially, thereby generating reaction thrust. While peristaltic induction may offer attractive efficiency and performance characteristics due to the high conductivity of the enclosed fluid (liquid metal), it does pose significant reliability and safety problems: the efficient liquid metals react chemically with most engineering materials--including water.

In summary, while all three types of electromagnetic propulsion pose significant technical problems, the internal duct design is simpler and probably more reliable than the others, and this work was commissioned specifically for a study of internal duct dc propulsion.

3.2 Performance Calculations

This section deals with specific items relating to performance evaluation, namely:

- choosing a primary measure of performance;
- separating the various interrelated parameters into dependent and independent variables;
- defining ranges of independent parameter variation.

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3.2.1 Measure of Performance

Since many interrelated parameters enter into the performance
calculations, we must decide upon the key or fundamental relationships
that we wish to examine: ultimately, what are we interested in comparing?
Useful thrust vs. input power at a fixed cruising speed? Or, cruising
range vs. stored energy as a function of hull size? Or, maximum speed
vs. electrode size as a function of magnetic field?

As mentioned earlier, internal duct E-M thrusters have an
inherently low power conversion efficiency, and this is perhaps a most
crucial measure of performance. Most of the results obtained in this
study, therefore, are given in terms of ideal[*] efficiency as speed
curves for several hull sizes. Additional curves show the effects of
conductivity enhancement (HCl seeding) as well as magnetic field intensity
upon efficiency. Finally, the last curves show overall input power and
thrust power as a function of speed for several hull sizes.

3.2.2 Parameter Specification

Clearly, we are not able to choose independent values for each
of the interdependent parameters that influence performance. If, for
example, we specify electrode voltage and current, then for a given duct
size, the thrust requirement (hull drag) determines magnetic field
intensity as a function of speed: all of these quantities cannot be
selected independently. Mathematically, we cannot exceed the degrees
of freedom that exist between the performance equations.

Our approach, therefore, has been to permit electrode voltage
and current to remain unspecified--they are dependent variables whose
values are determined as a consequence of other parameter selections.
Since dc electrical power sources may be configured to match or nearly

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[*]Ideal in that electrolysis and hydrodynamic thruster losses are not
included (see Sections 4.2, 4.9, and A-1). Magnetic excitation loss,
either in terms of ohmic power dissipation (conventional magnet) or
refrigeration power (superconducting magnet) are also not included in
these calculations.

match most power requirements (e.g., 10 volts at 200 amps or 200 volts at 10 amps), this is a reasonably prudent engineering approach.*

The following parameters are treated as independent variables and must be specified:

- magnetic field
- electrode length
- cross-sectional area of duct
- number of ducts
- fluid conductivity

3.2.3 Range of Parameter Variation

Lastly, the ranges of variation must be specified, and here again we are guided by practical engineering limitations rather than theoretical speculation. All five of the independent variables listed above are related in the sense that choosing one places practical constraints on the others. An extremely high, uniform magnetic field, for instance, cannot be established throughout a large volume. Duct dimensions must reflect this: we cannot seriously discuss duct lengths of hundreds of meters, nor can be consider cross-sectional areas greater than one or two square meters. The following ranges, therefore, were defined and specific calculations made for various combinations of these values:

Variable	Range
● Duct Area	0.25, 0.5, 1 square meters
● Electrode Length	2, 10 meters
● Magnetic Field	0.5, 2, 5 Tesla (2 to 5 T implies superconducting excitation)

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*It should be noted, however, that designing an actual thruster system may require several iterations wherein the thruster performance equations first determine an approximate power source; then actual power source terminal characteristics - such as would be available from various combinations of, say, submarine batteries - could be used for design modifications. An important part of this iteration would be variations in the aspect ratio, as discussed later in Section 4.9.

- Number of Ducts .... 1, 4, 9, 16 (Each of the multiple ducts has the same cross-sectional area and electrode length. That is, one duct is not subdivided into multiple channels such that the total thrust area remains constant, but rather the total duct area increases by factors of 4, 9, and 16.)

- Conductivity .... 4 mhos, the nominal conductivity of seawater. The effects of seeding with HCL were calculated by doubling the conductivity to 8 mhos. This corresponds to a 0.5% by weight solution of HCℓ.

4.0 THRUSTER PERFORMANCE CALCULATIONS

4.1 Thruster Model

Water-jet thrusters work on the principle of momentum conservation: an increase in stream momentum from intake to discharge produces a net reaction thrust. As shown in the pictorial sketch of Fig. 2, the electromagnetic thruster is composed of three sections: 1) converging inlet nozzle; 2) electrode-magnetic field region; 3) converging discharge nozzle. Definitions of the electrode (or pumping) region dimensions are also given in Fig. 2.

The converging inlet nozzle provides a smooth transition from the free-stream region immediately ahead of the intake to the internal pumping region, and if we assume that at a fixed uniform cruising speed, v_s^*, the inlet area is just large enough to provide the volumetric flow required, then no power will be expended drawing water into the inlet from ahead of the inlet plane. Matching the converging inlet opening to cruising speed also has the advantage of minimizing the thruster's displacement drag in that incident flow is directed into the duct, rather than around it.

The electrode or active pumping region consists of parallel electrodes mounted along the channel walls with a magnetic field established normal to the current-flow direction. The rectangular cross-sectional area, A_p, is assumed to be constant throughout the electrode region.

The third and last section of the thruster is the converging nozzle, which directs flow from the pumping region to a discharge orifice.

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*See Nomenclature for symbol definitions.

4.2 Assumptions

In the calculations that follow, the following assumptions have been invoked unless otherwise noted:

- Thrust = velocity-squared drag

    The thrust required to maintain a fixed speed is equal to total drag. Total hull drag, we assume, may be described by one equivalent drag coefficient that includes both frictional and displacement components. Total drag is then proportional to velocity squared times the cross-sectional area (see Fig. 1 for curves of drag vs. speed. Also see Appendix 1 for a comparison with more accurate drag calculations).

- Negligible thruster drag

    Based upon a thruster design that utilizes a converging inlet nozzle, this is a reasonable assumption (see Appendix 1 for details).

- Negligible electrode-electrolysis losses (electrolysis voltage << pump voltage). The validity of this assumption depends to a large extent upon the aspect ratio, see Section 4.9.

- Nozzle inlet velocity = relative velocity between submersible and surrounding water

- Absence of cavitation within the inlet nozzle

    A convergent inlet nozzle forces a static pressure drop with respect to the inlet or free stream pressure. Assuming no cavitation is equivalent to assuming that the submersible depth is great enough such that the absolute pressure at the pump inlet (P_{in}) is greater than the vapor pressure of water (several psia).

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4.3 Thruster Performance Equations

4.3.1 Thrust and Velocity Relationships

(See Fig. 2 and the Nomenclature for symbol definitions.)
From the conservation of momentum, we have

thrust = outlet momentum − inlet momentum

or

$$T = \dot{m}(v_{out} - v_{in}) \tag{1}$$

where the main flow, $\dot{m}$, is given by

$$\dot{m} = \rho v_p A_p = \rho v_s A_{in} = \rho v_{out} A_{out} \tag{2}$$

Combining these two equations, we can express thrust as

$$T = \dot{m} \, v_p \left(\frac{A_p}{A_{out}} - \frac{v_s}{v_p} \right) = \rho A_p v_p^2 \left(\frac{A_p}{A_{out}} - \frac{v_s}{v_p} \right) \tag{3}$$

But thrust developed by the duct is equal to the submersible's drag
force

$$D = (C_D A_s \rho/2) v_s^2 = K_s v_s^2 \tag{4}$$

or

$$T = K_s v_s^2 \tag{5}$$

where $K_s = C_D A_s \rho/2$

Combining Eqs. (3) and (5), we get for k_1 the ratio of internal duct
velocity to inlet velocity

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$$k_1 = \frac{v_p}{v_s} = \frac{A_{out}}{2A_p} \left[1 + (1 + 4K_s/\rho A_{out})^{1/2} \right] \tag{6}$$

Also, note that from Eq. (2), $v_p A_p = v_s A_{in}$, so that $k_1 = A_{in}/A_p$ = ratio of inlet area to area of pumping region.

4.3.2 Pressure Relationships (neglecting hydrodynamic losses)

Inlet

Applying the Bernoulli Equation, we have

$$P_o + 1/2 \, \rho \, v_s^2 = P_{in} + 1/2 \, \rho \, v_p^2 \tag{7}$$

Rearranging and using the pressure differential ΔP_{in} yields

$$\Delta P_{in} = P_o - P_{in} = 1/2 \, \rho \, (v_p^2 - v_s^2)$$

or

$$\Delta P_{in} = 1/2 \, \rho \, v_p^2 \, (1 - (v_s/v_p)^2) \tag{8}$$

Exit Nozzle

Again, applying the Bernoulli Equation we have

$$P_z + 1/2 \, \rho \, v_p^2 = P_o + 1/2 \, \rho \, v_{out}^2 \tag{9}$$

then the nozzle pressure drop is

$$\Delta P_z = P_z - P_o = 1/2 \, \rho \, (v_{out}^2 - v_p^2)$$

or using Eq. (2) to express v_{out} in terms of v_p, we get

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$$\Delta P_z = 1/2 \; \rho \; v_p^2 \left[\left(\frac{A_p}{A_{out}} \right)^2 - 1 \right] \tag{10}$$

Pump

$$\text{pump head} = \Delta P_p = P_z - P_{in} = \Delta P_z + \Delta P_{in} \tag{11}$$

Substituting from Eqs. (8) and (10):

$$\Delta P_p = 1/2 \; \rho \; v_p^2 \left[\left(\frac{A_p}{A_{out}} \right)^2 - \left(\frac{v_s}{v_p} \right)^2 \right] \tag{12}$$

But from Eq. (6), $v_p = k_1 v_s$ and we can express the pump head in terms of the relative velocity, v_s:

$$\Delta P_p = 1/2 \; \rho \; v_s^2 \left[k_1^2 \left(\frac{A_p}{A_{out}} \right)^2 - 1 \right] \tag{13}$$

This is the pump head or pressure differential that must be generated electromagnetically in order to sustain the relative velocity v_s. If the pump pressure satisfies this requirement, then thrust = drag at speed v_s.

4.3.3 Electromagnetic Pressure

Neglecting electric and magnetic fringe effects near the electrode edges, we have:[*]

$$\Delta P_p = JBL = \sigma BL \left(\frac{V}{2b} - v_p B \right) \tag{14}$$

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[*]See Appendix A-2 for a derivation of this equation.

4.3.4 Voltage and Current

Substituting for ΔP_p from Eq. (13) and solving for the electrode voltage, V, we get

$$V = \frac{\rho v_s^2 bf}{\sigma BL} + 2Bv_s k_1 b \qquad (15)$$

where $f = [(k_1 A_p / A_{out})^2 - 1]$.

Now we can use Eqs. (13), (14), and (15) to solve for the current density, J, and hence the total current, I (= 2aIJ):

$$I = 2aL\sigma \left(\frac{V}{2b} - v_p B\right)$$

$$I = \frac{a \rho v_s^2}{B} ((k_1 A_p / A_{out})^2 - 1)$$

or

$$I = a \rho v_s^2 f/B \qquad (16)$$

where f is the dimensionless parameter given above.

4.3.5 Efficiency

Pumping efficiency, η, is the ratio of thrust power to electrical input power

$$\eta = T v_s / IV \qquad (17)$$

Substituting from Eqs. (5), (15), and (16), we have

$$\eta = 2K_s / [A_p \rho f k_1 (f/(2k_1^2 N) + 1)] \qquad (18)$$

where again $f = [(k_1 A_p / A_{out})^2 - 1]$ and we introduce the interaction parameter N, a measure of the interaction between electromagnetic and inertial body forces in the fluid, defined as

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$$N = 8B^2 L/c \, v_p \tag{19}$$

Efficiency may also be expressed in terms of the propulsive and electrical efficiencies, which are defined, respectively, as

$$\eta_p = \frac{Tv_s}{W_f} \tag{20}$$

$$\eta_e = \frac{W_f}{I \cdot V} \tag{21}$$

where the fluid power, $W_f = v_p \Delta P_p A_p$.

Substituting from the previous expressions for these quantities, we have

$$\eta = \eta_p \cdot \eta_e \tag{22}$$

with

$$\eta_p = \frac{2}{k_1 A_p / A_{out} + 1} \tag{23}$$

$$\eta_e = \left[1 + \frac{1}{2N} \left(\frac{A_p}{A_{out}} \right)^2 - \frac{1}{k_1^2} \right]^{-1} \tag{24}$$

From Eq. (24) we note that efficiency increases as the nozzle discharge area A_{out} approaches the pump area, A_p. To maximize overall efficiency, then, we set $A_{out} = A_p$. This corresponds to the maximum cross-sectional discharge area consistent with no nozzle cavitation, and the governing equations become:

•• Velocity ratio:

$$k_1 = \frac{v_p}{v_s} = \frac{A_{in}}{A_p} = (1/2) \left[1 + (1 + 4K_s / \rho A_p)^{1/2} \right] \tag{25}$$

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- Electrode voltage:

$$V = 2Bv_s k_1 b \; (f/(2k_1^2 N) + 1) \qquad\qquad (26)$$

where $f = (k_1^2 - 1)$ and

N = interaction parameter defined above.

- Electrode current:

$$I = a \, \rho \, v_s^2 \; f/B \qquad\qquad (27)$$

- Efficiency:

$$\eta_p = 2/(k_1 + 1)$$
$$\qquad\qquad (28)$$
$$\eta_e = [1 + \frac{1}{2N} \, (1 - k_1^{-2})]^{-1}$$

where overall efficiency $\eta = \eta_p \cdot \eta_e$.

Equations (25) through (28) provide a basis for evaluating performance of a single duct propulsion system.

4.3.6 Multiple Duct Performance

The single duct equations may be adapted to multiple duct analysis simply by altering the thrust coefficient, K_s. Let $K_s' = K_s/n =$ thrust requirement for each of n ducts. Then we get the following multiple duct relationships:

- per duct velocity coefficient:

$$k_1' = v_p'/v_s = (1/2) \; [1 + (1 + 4K_s'/\rho \, A_p')^{1/2}] \qquad\qquad (6')$$

where $A_p' =$ area per duct

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• electrode voltage:

$$V' = 2Bv_sk_1'b'(f'/(2k_1'^2 \cdot N') + 1) \tag{26'}$$

$$\text{where } f' = k_1'^2 - 1, \; N' = \sigma B^2 L/\rho v_p'$$

• electrode current:

$$I' = a' \rho v_s^2 f'/B \tag{27'}$$

• efficiency:

$$\eta' = [2/(k_1' + 1)] \cdot [1 + (2N')^{-1} (1 + k_1'^{-2})]^{-1} \tag{28'}$$

Suppose, then, that we compare the performance of n parallel ducts to a single duct where the combined parallel duct pumping area is equal to that of the single duct (that is, $A_p' = A_p/n$, $a' = a/\sqrt{n}$, $b' = b/\sqrt{n}$) then:

$$k_1' = k_1, \; f' = f, \; N' = N$$

$$I' = I/\sqrt{n}, \; V' = V\sqrt{n}$$

$$\eta' = \eta$$

And we see that there is no change in efficiency. If, however, we retain the same area per duct, but add n parallel ducts ($A_p' = A_p$, $a' = a$, $b' = b$) then the velocity ratio k_1' is given by:

$$k_1' = (1/2) [1 + (1 + 4K_s/\rho nA_p)^{1/2}] \tag{29}$$

In this case, $k_1' < k_1$, due to the "n" factor appearing in the denominator. Also, we find $f' < f$ and $N' > N$. Substituting these inequalities into Eq. (28'), we find that the multiple duct efficiency increases ($\eta' > \eta$). Thus, increasing the total thrust area by adding

parallel ducts decreases the inlet-to-pump velocity ratio and increases efficiency.

4.4 Efficiency Plots

Efficiency plots based upon Eq. (28') are given in Figs. 3 to 26, where calculated, ideal efficiencies (based upon the assumptions discussed earlier) are plotted as a function of submersible velocity, v_s. Each plot contains a set of curves obtained using drag character- istics for three hull sizes, as given in Fig. 1. For each hull size, single duct efficiency is plotted along with several curves showing the effect of four, nine, and sixteen multiple ducts. The figures are arranged in three groups of eight, each group corresponding to ascending magnetic field intensities of 0.5, 2, and 5 Tesla, respectively. Within each group, electrode lengths of 2 and 10 meters are combined with duct areas of .25, .5, .75 and 1.0 square meters. As may be seen from the curves, efficiencies vary from less than .01 to over 99 percent. It must be emphasized that these plots are useful for comparing and indicating sensitivity of ideal performance to parameter variations--all of these combinations are not necessarily realistic. Sixteen thrusters, with a combined cross-sectional area of sixteen square meters propelling a one- meter diameter hull does not, for instance, represent a realistic config- uration. But one such thruster propelling this size hull may be entirely feasible.

Not surprisingly, these plots show that efficiency improves as electrode length, magnetic field, duct area, and the number of ducts increase. Low thrust requirements due to less drag (smaller hull sizes) also improves efficiency.

4.5 Conductivity Enhancement: HCl Seeding

As indicated by the performance equations derived in Section 4.3, electrical losses diminish and thrust efficiency increases as fluid conductivity increases. The conductivity of seawater, which has a

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nominal value of about 4 mhos/m, increases significantly with the
addition of a small percentage of hydrochloric acid (HCL). Neglecting
possible increases in electrode corrosion rates, conductivity enhancement
in a thruster may be accomplished by injecting HCL into the inlet nozzle
where mixing and diffusion tends to create a uniform mixture prior to
the electrode region. While from an efficiency consideration there is
strong incentive to inject large amounts of HCL, the benefits of con-
ductivity enhancement must be balanced against the limitations of solute
storage capacity on board the submersible. As a compromise between these
conflicting requirements, we have arbitrarily chosen a 0.5% solution
(.14 molar concentration of HCL), which effectively doubles the bulk
conductivity of seawater from 4 mhos/m to 8 mhos/m.[6] Figs. 27-29 show
the effects of conductivity enhancement upon ideal efficiencies for
full, third, and tenth size hulls, respectively. Note that the basic
thruster-duct size (2 m x .25 m^2) is identical in all three cases but
that the number of ducts attached to each hull varies from: sixteen
ducts, full-size hull; nine ducts, third-size hull; four ducts,
tenth-size hull. These configurations are more representative of actual
design possibilities than, say, a fixed number of ducts for all three
sizes.

Seeding flow rates shown on the plots vary from 9400 gpm at
20 knots, full-size hull to 71 gpm at 2 knots, tenth-size hull. In
general, efficiency increases somewhat less than the factor of two
increase in conductivity.

4.6 Sensitivity of Efficiency to Magnetic Field Strength

Ideal efficiencies for the three hull sizes and duct combinations
(described in Section 4.5) are plotted in Fig. 30 for a fixed speed of
5 KTS and magnetic field variations from .5 to 5 Tesla. Although data for
the plots are contained in the basic efficiencies curves given in
Figs. 2 to 26, Fig. 30 more clearly illustrates the magnetic field
dependence where an order of magnitude change in field intensity yields
efficiency changes of nearly two orders of magnitude. As shown in the

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figure, conventional excitation is limited to less than 2 Tesla due to saturation of ferromagnetic materials used in magnetic circuits.

4.7 Power Requirements Vs. Speed

Total thrust power and electrical input power curves over a speed range of 2 to 20 knots are shown in Fig. 31 for the duct-hull combinations described in Section 4.5. These plots, which are based on a 2 T magnetic field intensity, underline the importance of low speed operation: doubling the cruising speed from 5 to 10 KTS, for instance, increases the total input power requirement by a factor of ten.

4.8 Efficiency as a Function of Interaction Parameter

The electrical and propulsive efficiency expressions given by Eqs. 28 and 29 may be greatly simplified by substituting some typical values for the interaction parameter.   Since the order of magnitude is determined mainly by the σ/ρ ratio ($\approx 4 \cdot 10^{-4}$) in the expression $N = \sigma B^2 h/\rho v_p$, we can safely assume $N \ll 1$, or N is less than .01. And if the velocity ratio k_1 is much greater than 1 ($k_1 \gg 1$), then from Eqs. 28 and 29 the overall efficiency $\eta < (.5)(2N)$ or $\eta < N$. That is, the efficiency is less than one percent. If, on the other hand, we assume $k_1 = 1+\epsilon$ where $\epsilon \ll 1$ (that is, the velocity differential $v_p - v_s = \epsilon v_s$), then

$$\eta_p' = \frac{2}{2 + \epsilon} = \frac{1}{1 + \epsilon/2} \approx 1 - \epsilon/2 \tag{30}$$

and

$$\eta_e' = \frac{1}{1 + (\epsilon/N)} \tag{31}$$

These expressions indicate that reasonable efficiencies may be achieved, provided that ϵ, the velocity differential, is of the same order as the interaction parameter, N. This is illustrated by the curve in Fig. 32.

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In addition to efficiency, though, a practical design must include other considerations such as: electrode voltage and current, and overall pump dimensions. While simply meeting the $\varepsilon \approx N$ criterion does not guarantee a practical design, the perfc nance expressions discussed here do indicate the sensitivity of overall efficiency to the v_p/v_s ratio.

4.9 Performance as a Function of Aspect Ratio

Although overall ideal efficiency is independent of aspect ratio, actual efficiency will depend to some extent on internal shape of the thruster ducts. This arises for two reasons:

- Viscous drag increases with wetted surface area. A square duct ($\alpha=1$), therefore has minimum viscous drag. (See Appendix 1 for a detailed discussion of duct drag.)

- Electrolysis voltage remains nearly constant over a wide range of operating current densities.[*] The ratio between this fixed voltage and the total electrode voltage, though, diminishes as the aspect ratio increases. That is, a duct cross section elongated in the direction of electrode separation requires a higher pump voltage and electrolysis losses become a small fraction of total electrical losses as the elongation increases.

Hence, we have two conflicting requirements, one for a unity aspect ratio in order to reduce viscous drag, the other for a high aspect ratio to minimize electrolysis losses. A careful trade-off analysis must be included in actual thruster designs.

A conventional copper-iron magnetic circuit will produce a more uniform field with reduced excitation losses as the aspect ratio increases. This, too, must be factored into the overall design.

_____

[*]The electrolysis potential at both the anodic and cathodic electrodes is ≈ 2 volts. Since this value is weakly dependent upon current density and strongly dependent on electro-chemical effects, it is realistic to assume a total electrolysis voltage of between five and ten volts.

5.0 GAS EVOLUTION THROUGH ELECTROLYSIS

Dissociation of water at the cathode discharges hydrogen gas along the electrode: two faradays of charge generates one mole (or 22.4 l of H_2 at STP) so that:

$$Q_{H_2} = \text{hydrogen discharge rate} = 5.210^{-3} \ I_{kA} \ 1/s \qquad (32)$$

This, of course, mixes with the thrust-exit stream of water and eventually bubbles to the surface. Using values for discharge flow and electrode currents associated, for example, with the full-hull size curves of Fig. 3, we have (at 20 KTS):

$$\text{thrust stream per duct} = 4.4 \times 10^3 \ 1/s$$

$$\text{electrode current/duct} = 104 \ kA$$

then

$$Q_{H_2} = .54 \ 1/s \qquad ,$$

and we find an extremely low hydrogen-to-water volume ratio. Evolved hydrogen gas along the electrode surfaces should not hinder electrode performance, nor present a noticeable bubble or foam wake at the surface. Also, it may be possible to construct a porous electrode that captures the evolved gases. Final verification of these conclusions and possibilities must be made experimentally.

6.0 PRELIMINARY DESIGN PROCEDURE

Designing an electromagnetic thruster requires many iterations through the performance equations in order to balance efficiency, physical size and magnetic field intensity with desirable electrical characteristics. The following procedure is offered as a plausible design approach:

(1) Using the known drag vs. speed requirements (K_s) for a particular submersible and assuming a fixed number of thruster ducts of area A_p, find the velocity coefficient, k_1 from Eq. 29. This determines the inlet-to-pump cross-sectional ratio (A_{in}/A_p).

(2) Using Eqs. 26', 27', and 28', calculate voltage, current, and efficiency at various combinations of the following parameters:

- aspect ratio (α)
- pump length (L)
- magnetic field (B)
- velocity (v_s)

(3) Repeat steps (1) and (2) over the range of interest for:

- drag
- cross-sectional area per thrust-duct
- total number of thrusters

This procedure may be refined by including:

- duct drag (see Appendix A-1)
- electrolysis voltage (see Section 4.9)
- electromagnetic fringe effects
- inlet/outlet hydrodynamic losses

(These last two items should remain second order effects and are not pursued here.)

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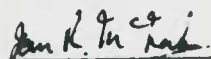
ACKNOWLEDGMENTS

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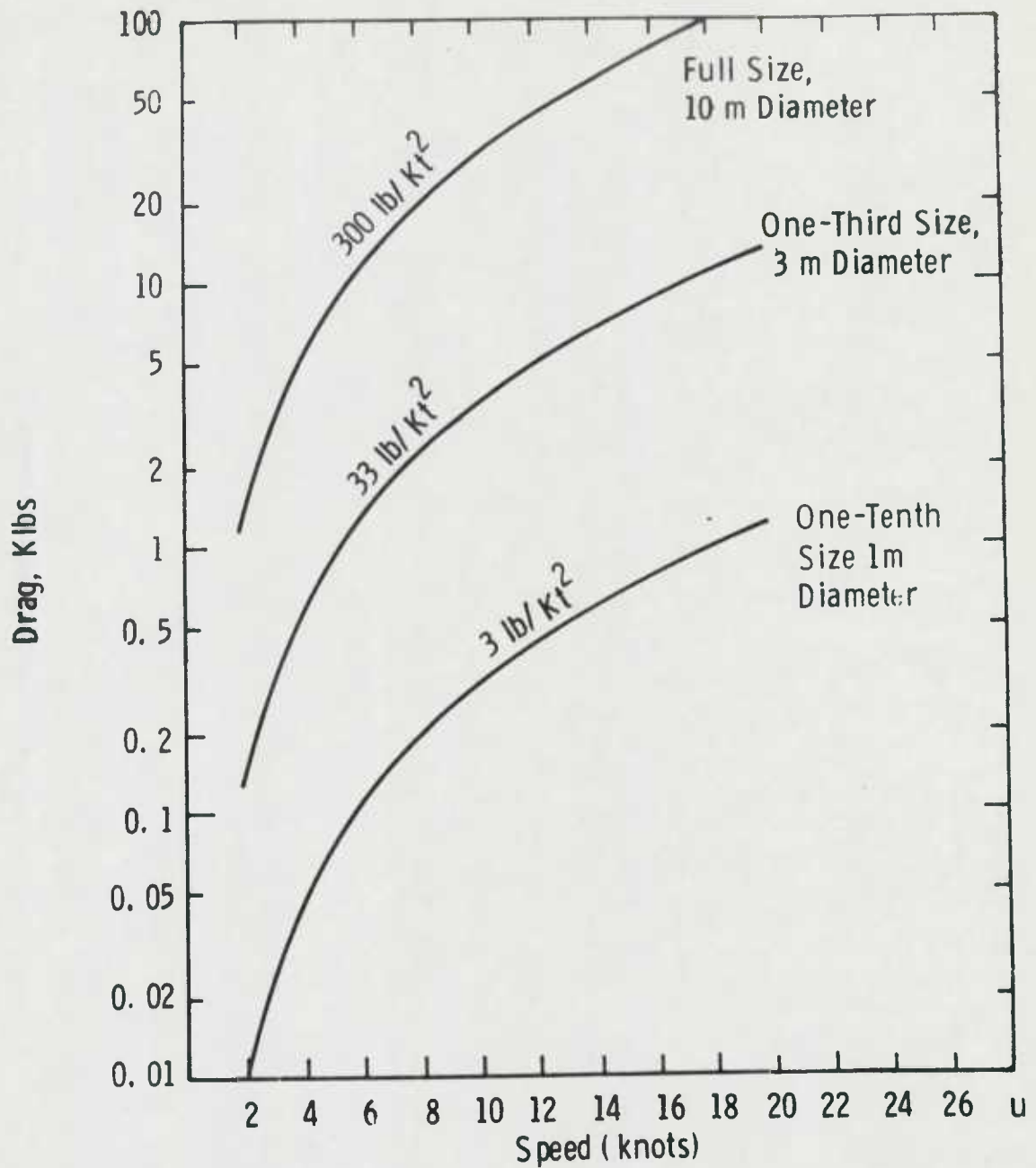
Fig. 1 — Drag characteristics used for performance calculations
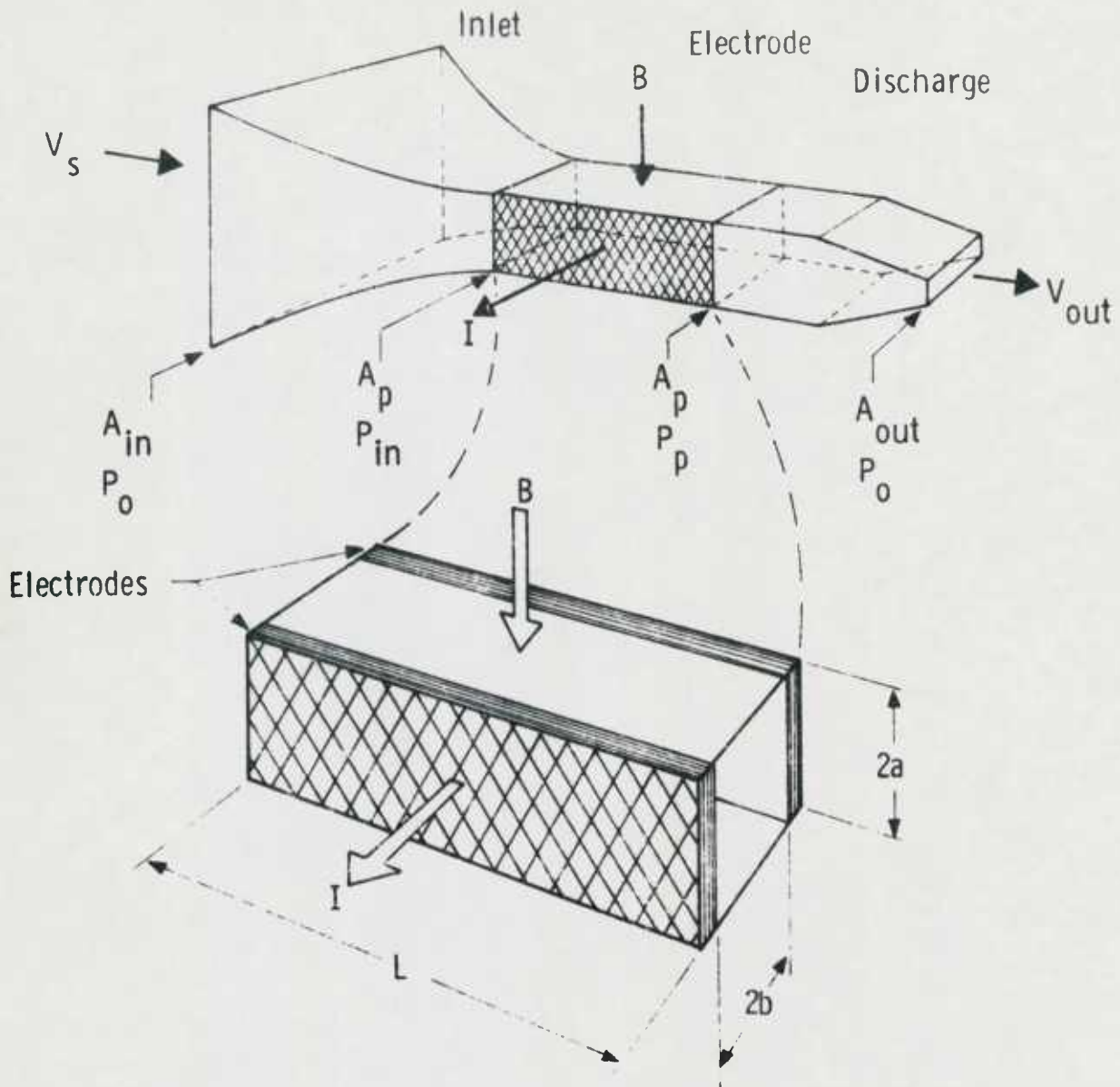
Fig. 2 — Pictorial diagram of internal duct thruster with converging inlet and outlet nozzles. Details of electrode region showing configuration used for duct analysis
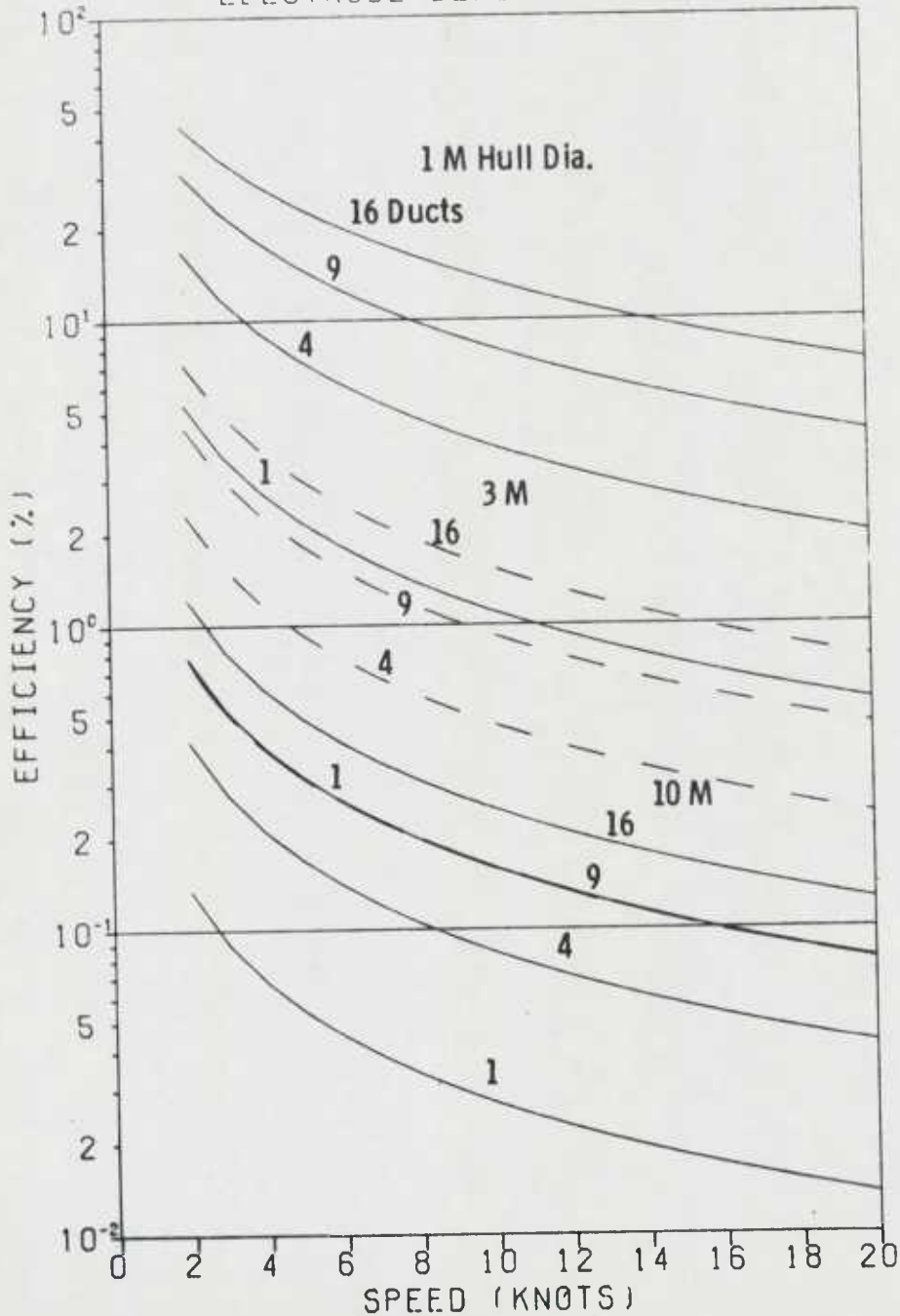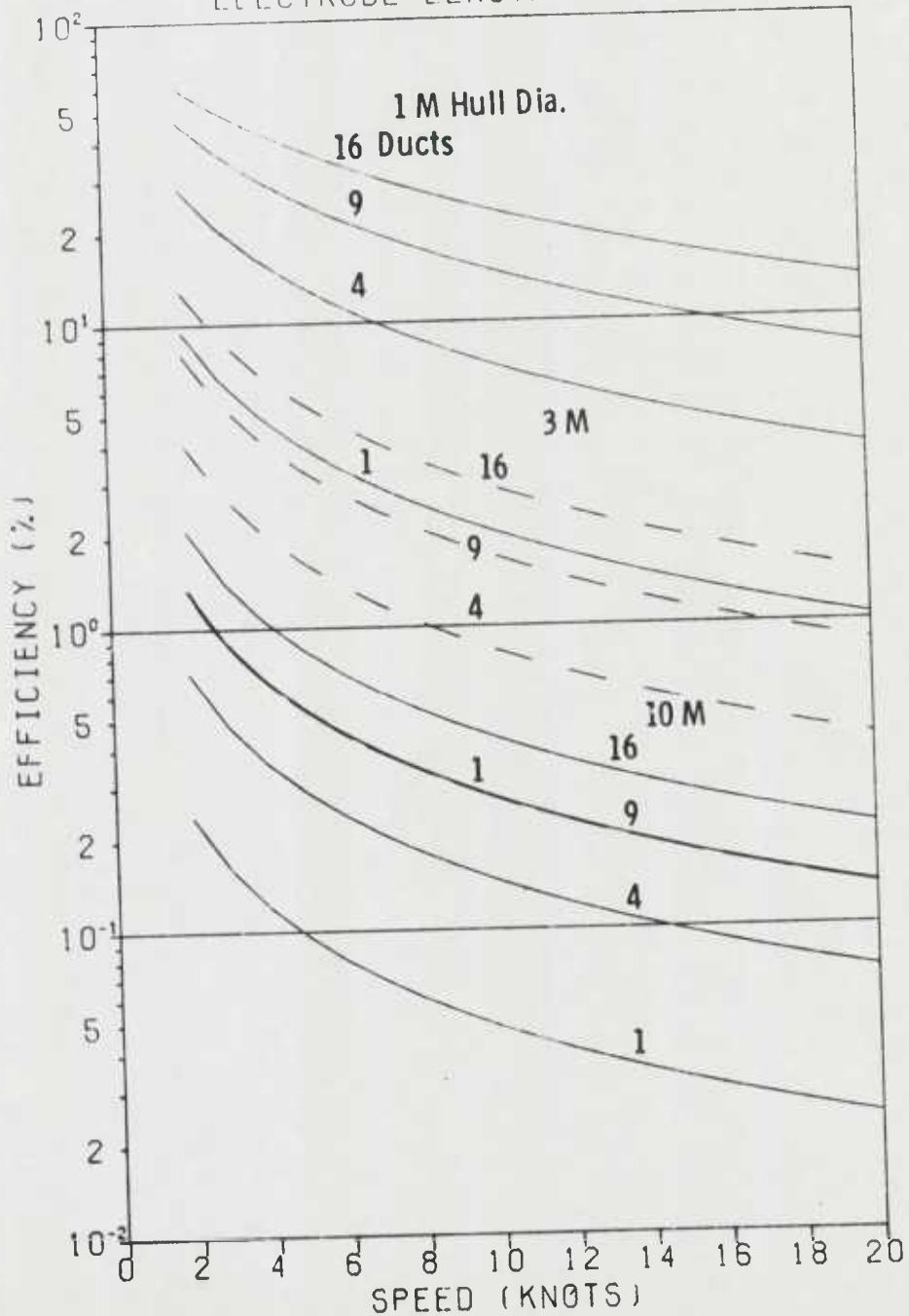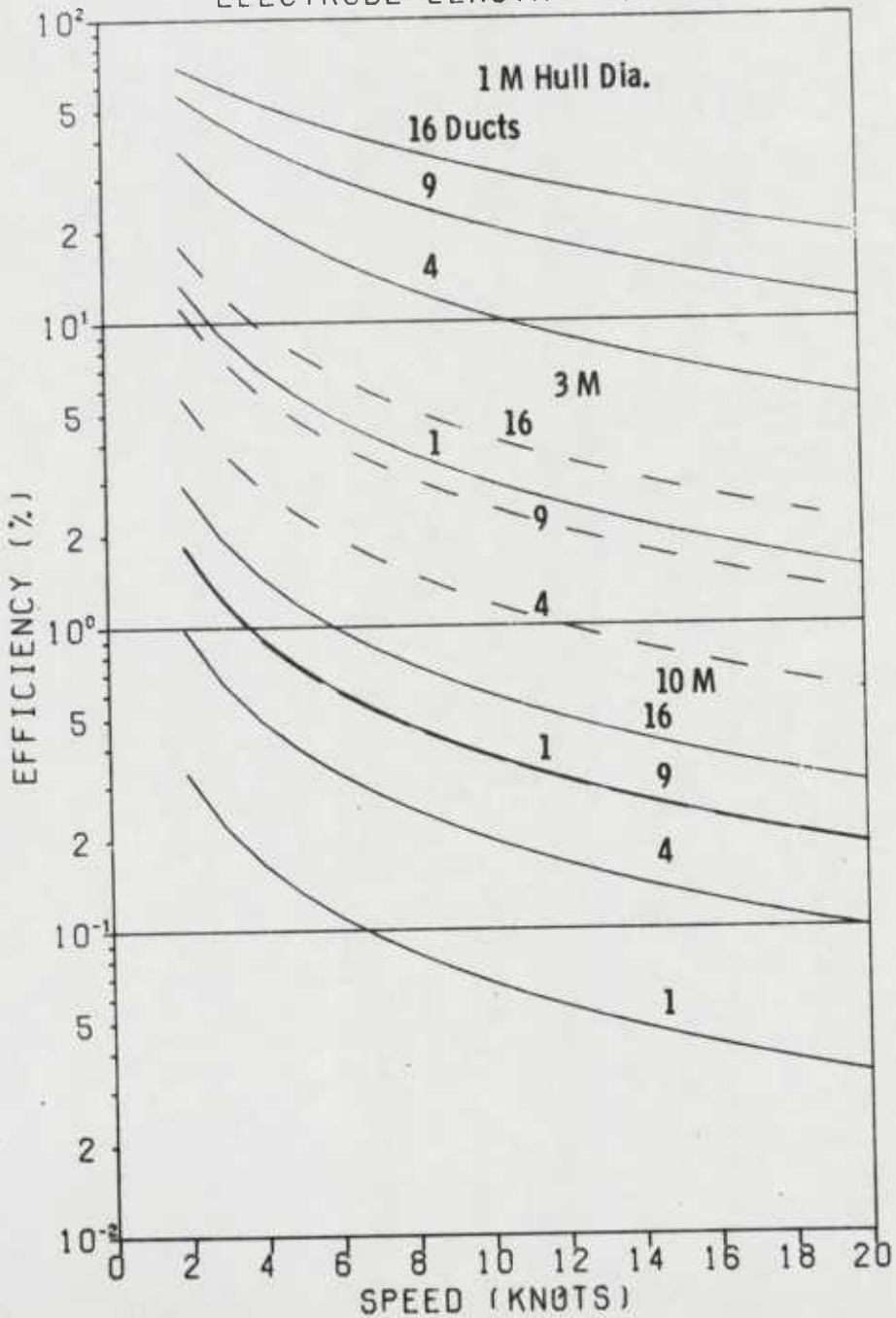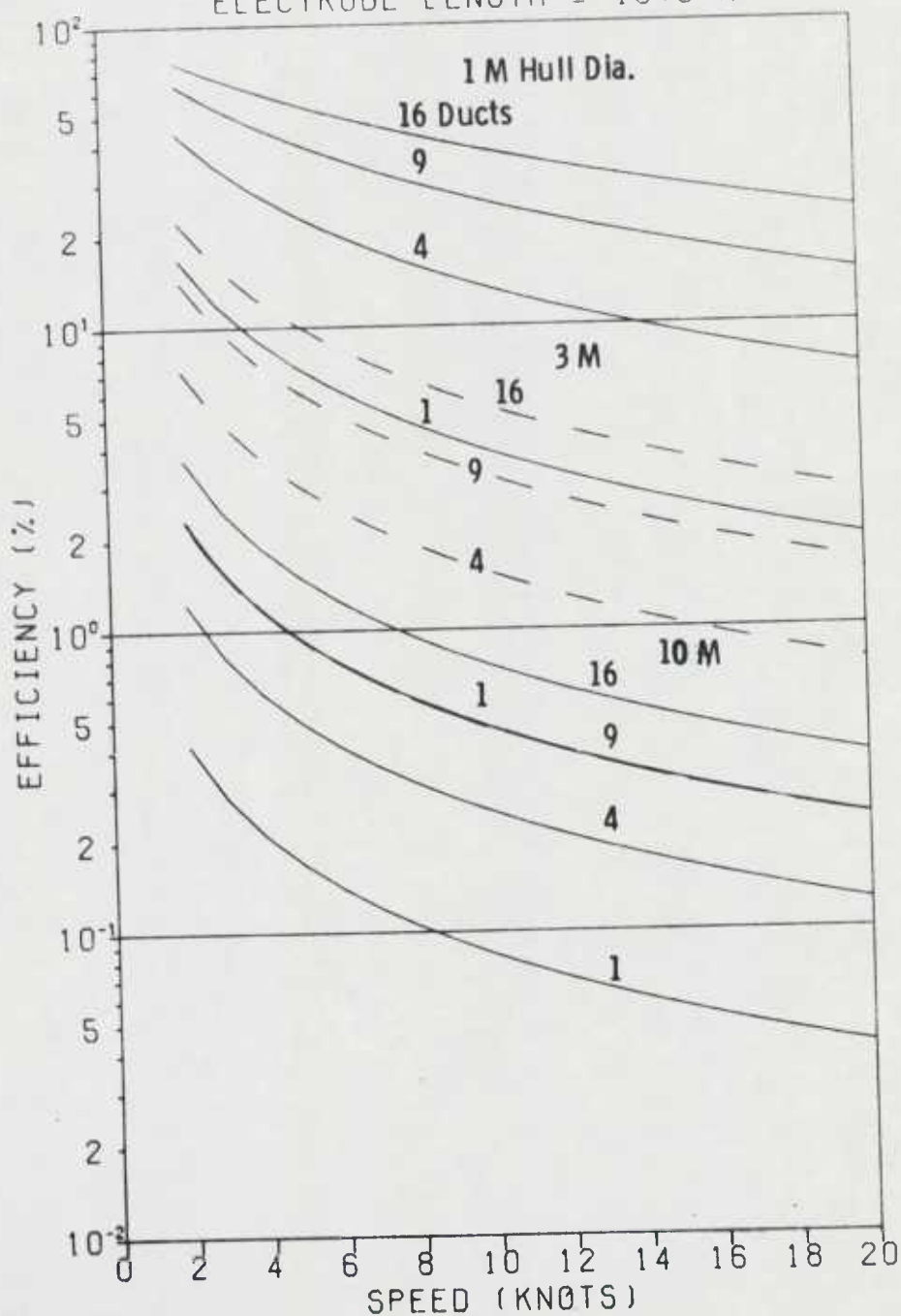
29

FIGURE 3

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD = 0.5 T
EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 2.0 M
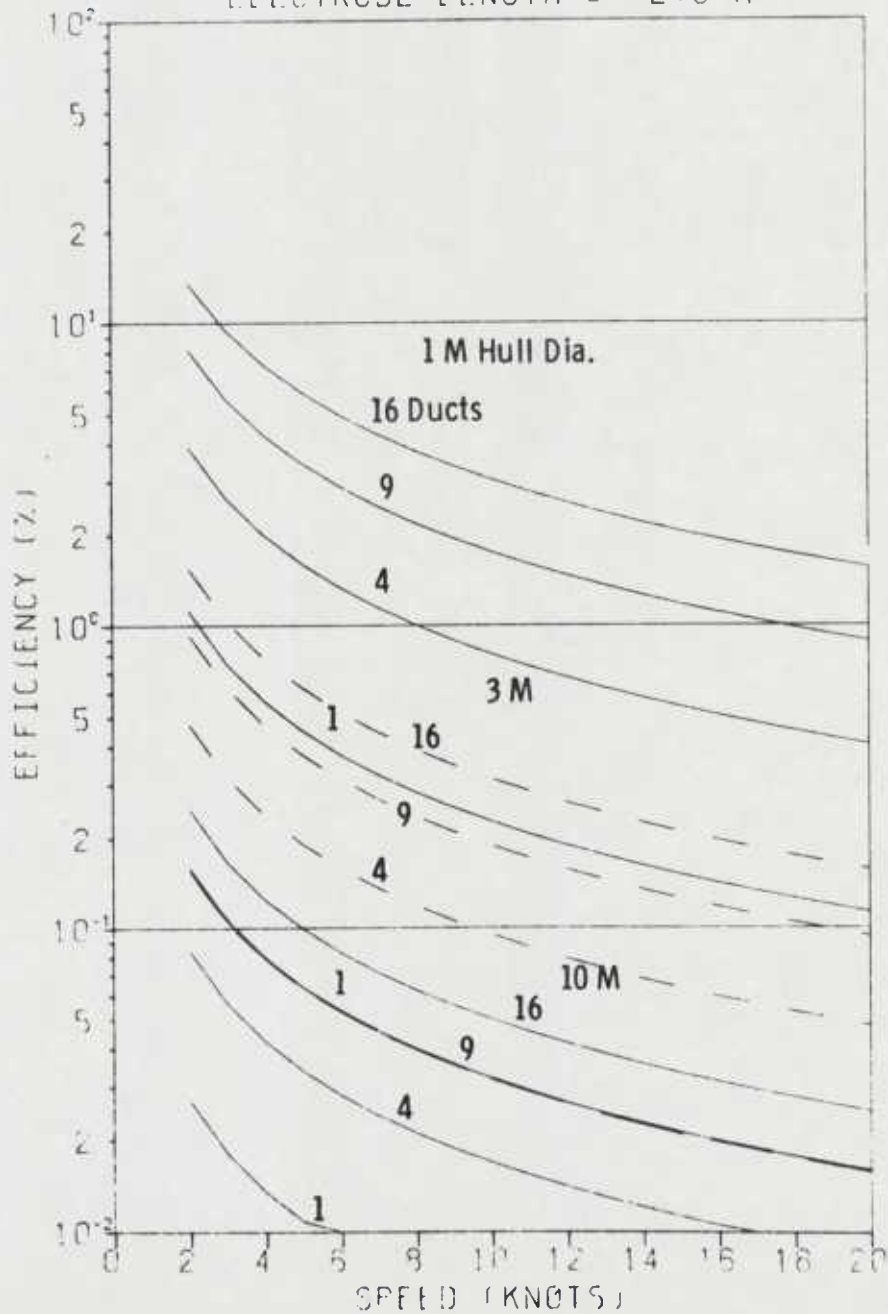
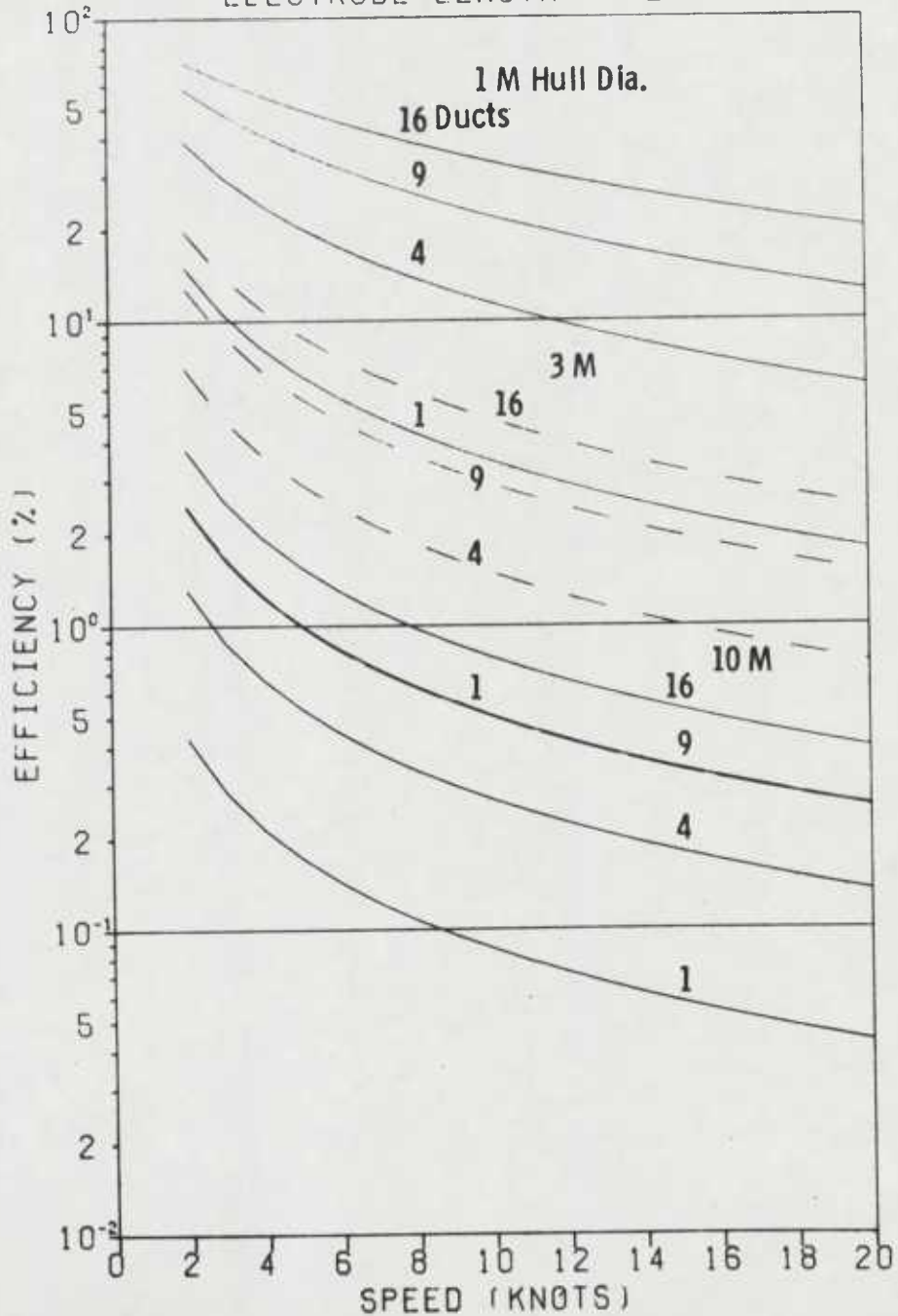FIGURE 4

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 0.5 T
EACH DUCT AREA = .50 SQ M
ELECTRODE LENGTH = 2.0 M
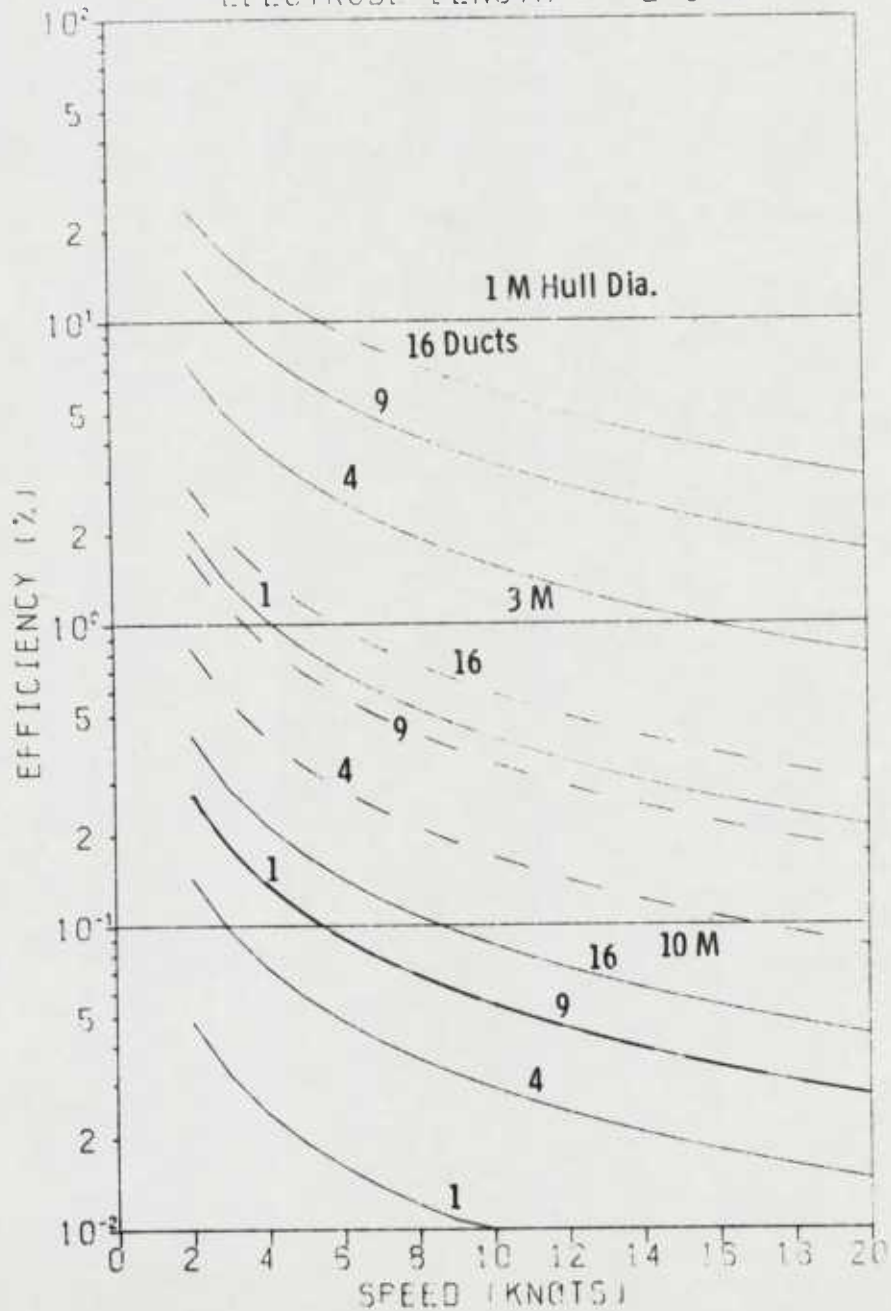
FIGURE 5

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 0.5 T
EACH DUCT AREA   = .75 SQ M
ELECTRODE LENGTH = 2.0 M

32
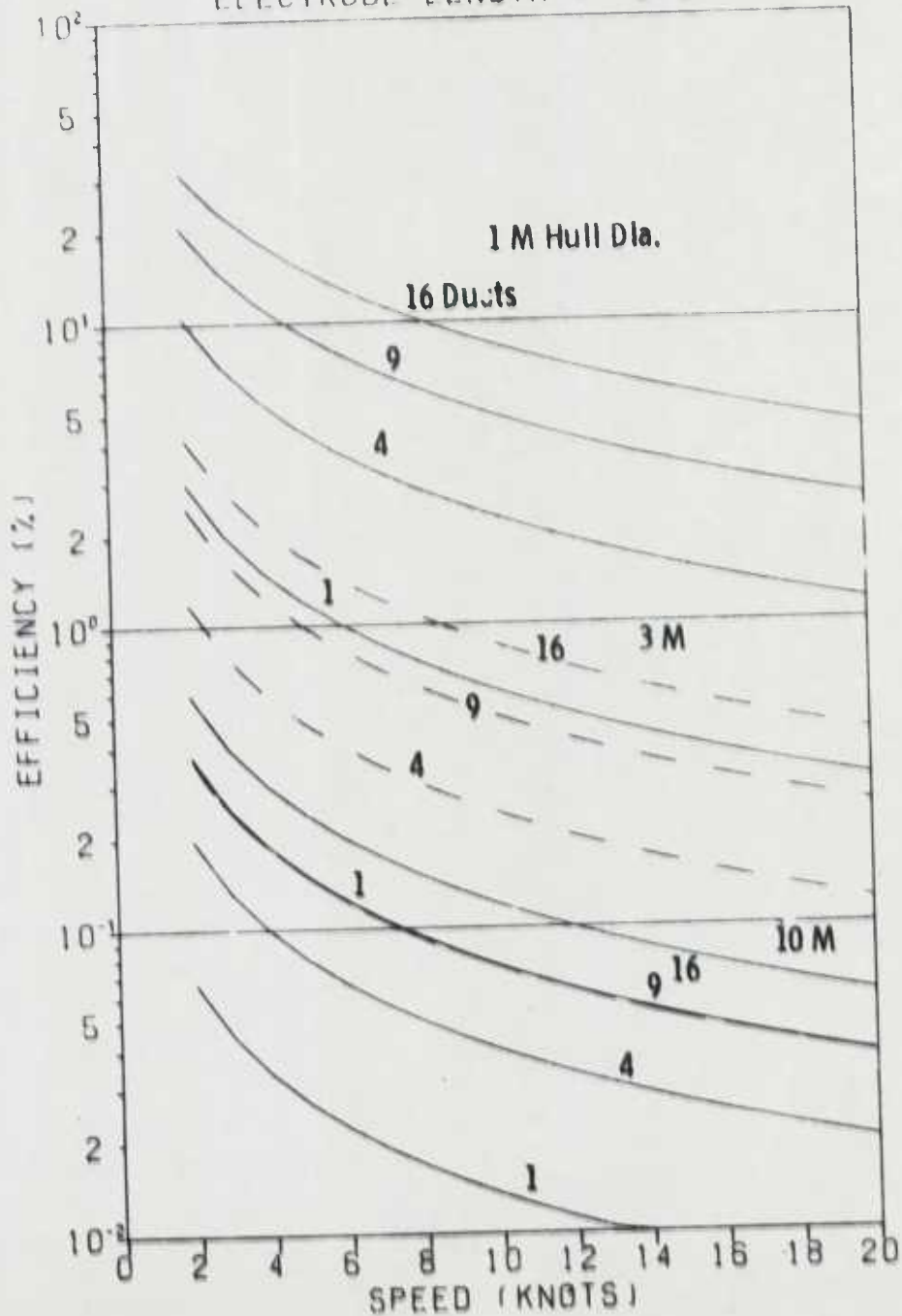
FIGURE 6

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD = 0.5 T
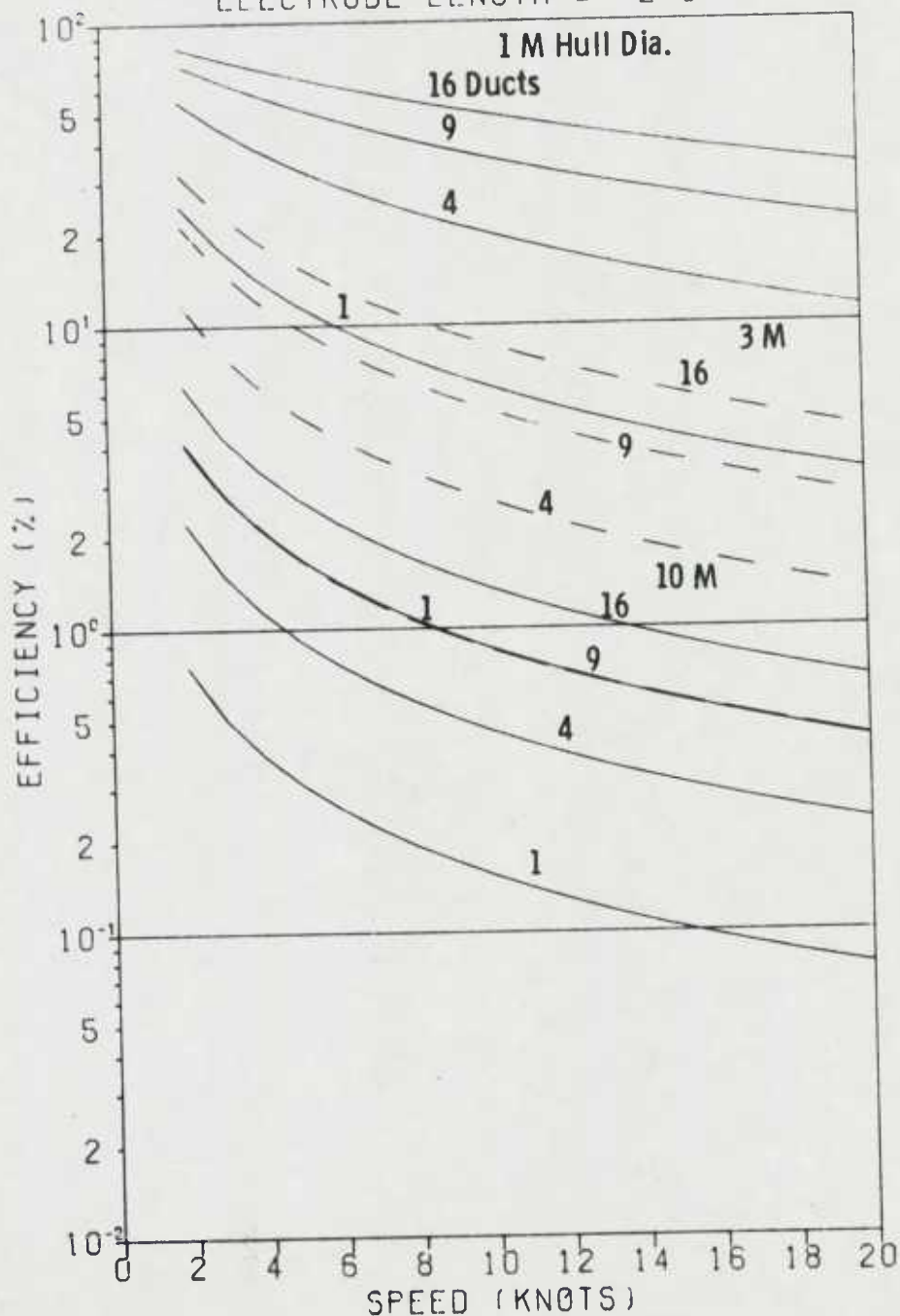EACH DUCT AREA = 1.0 SQ M
ELECTRODE LENGTH = 2.0 M
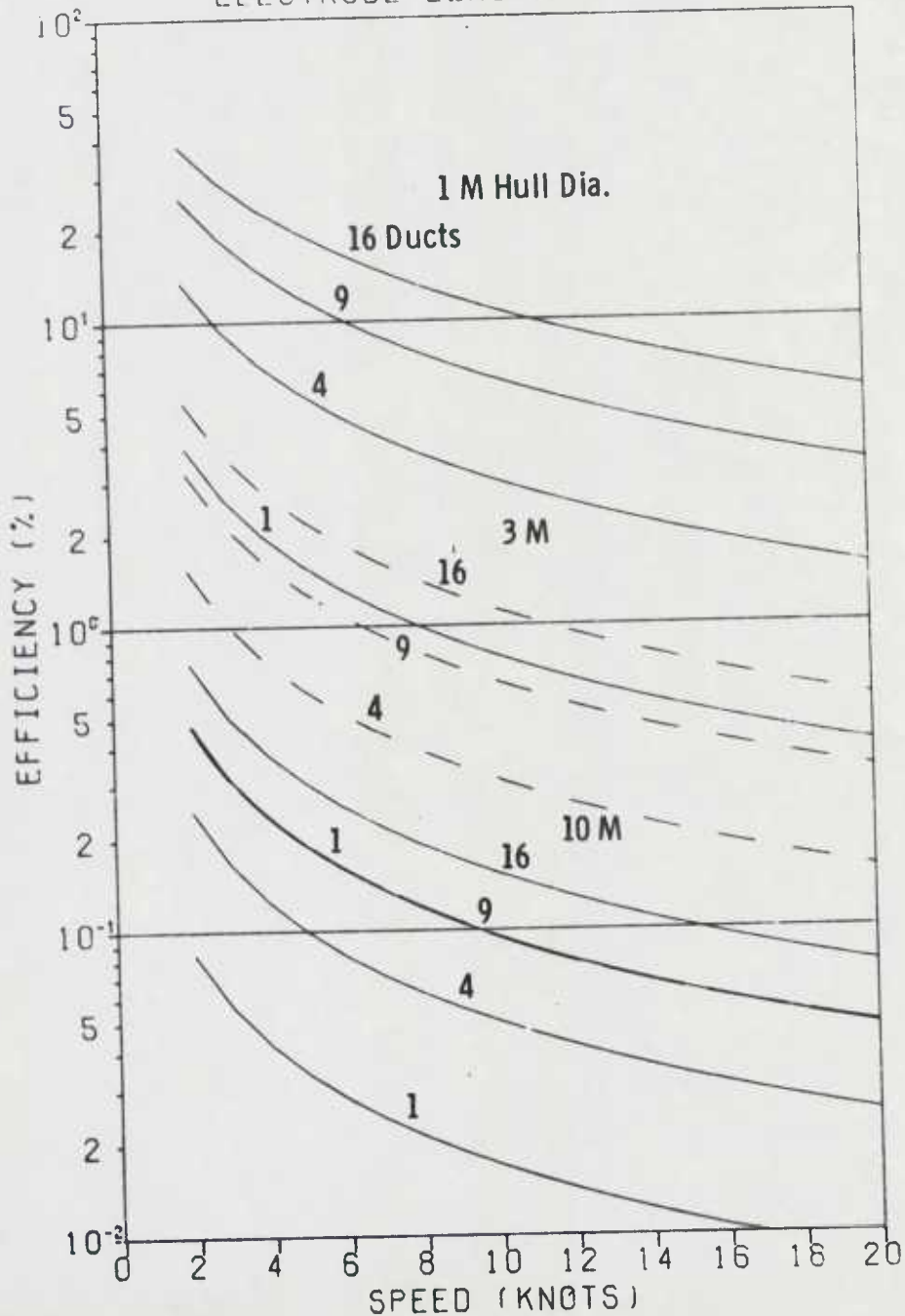
FIGURE 7

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD   = 0.5 T
EACH DUCT AREA   = .25 SQ M
ELECTRODE LENGTH = 10.0 M

FIGURE 8

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 0.5 T
EACH DUCT AREA   = .50 SQ M
ELECTRODE LENGTH = 10.0 M

FIGURE 9

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 0.5 T
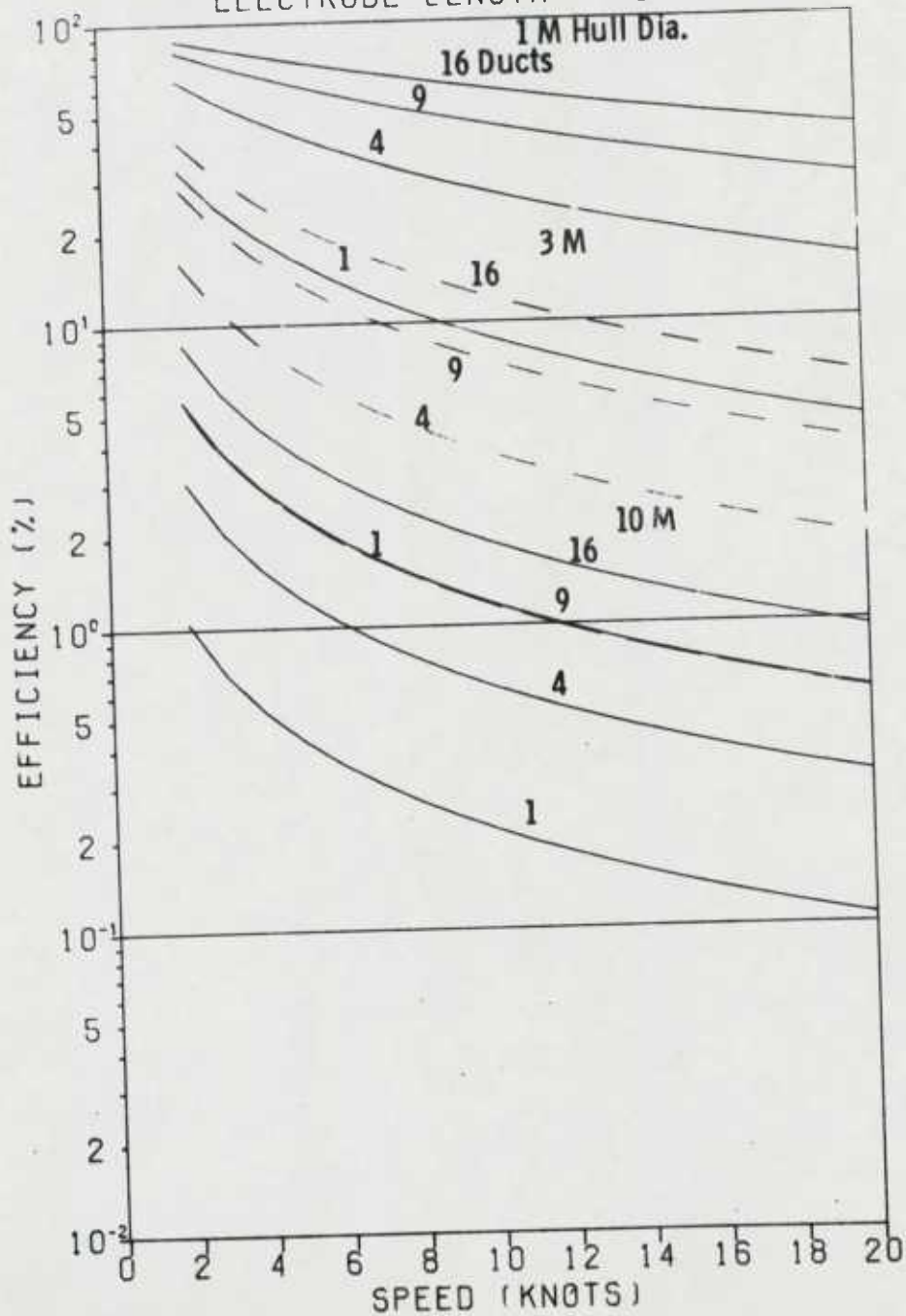EACH DUCT AREA   = .75 SQ M
ELECTRODE LENGTH = 10.0 M

FIGURE 10

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD = 0.5 T
EACH DUCT AREA = 1.0 SQ M
ELECTRODE LENGTH = 10.0 M

FIGURE 11

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
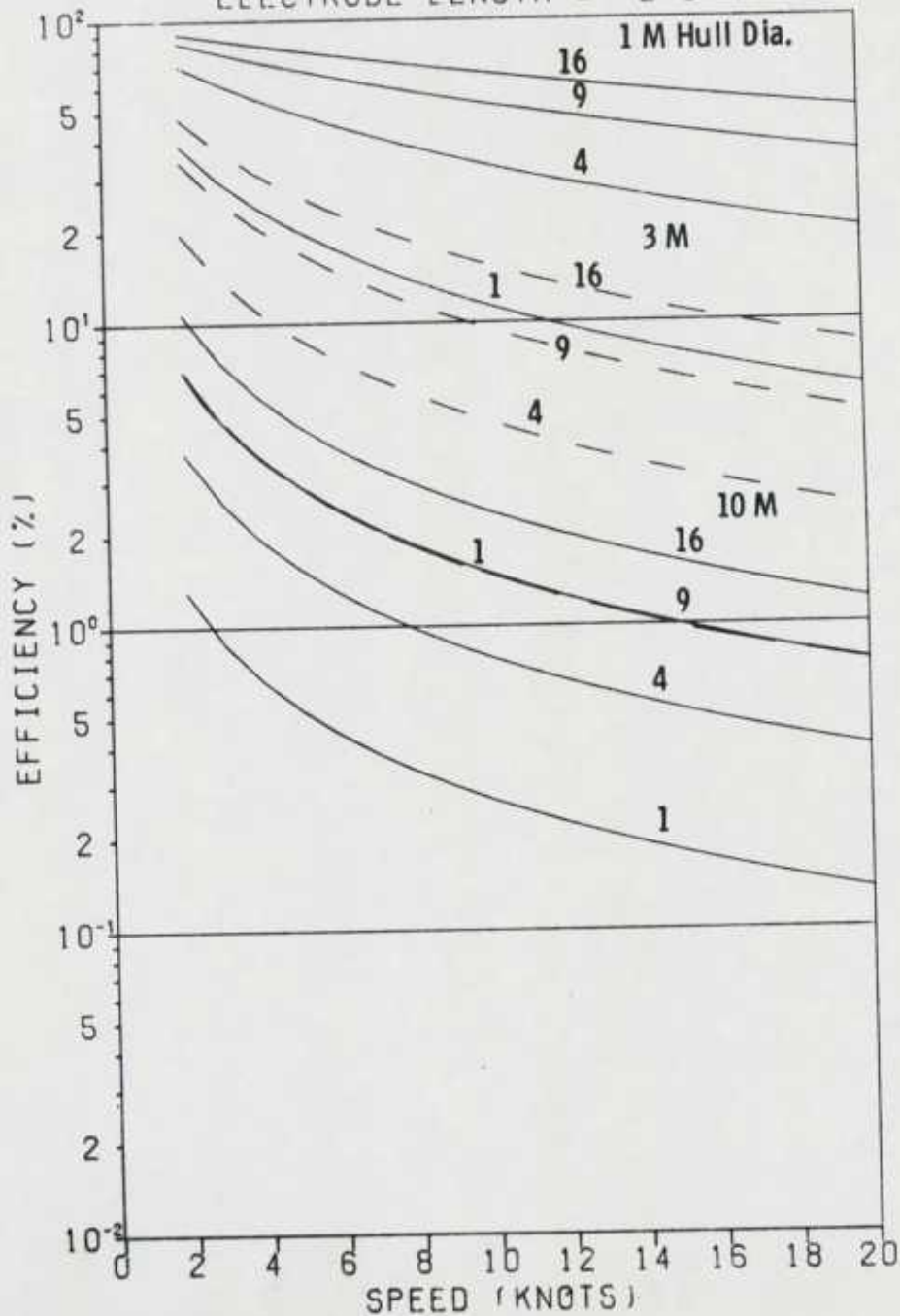EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 2.0 M

FIGURE 12

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 2.0 T
EACH DUCT AREA. = .50 SQ M
ELECTRODE LENGTH = 2.0 M

FIGURE 13

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 2.0 T
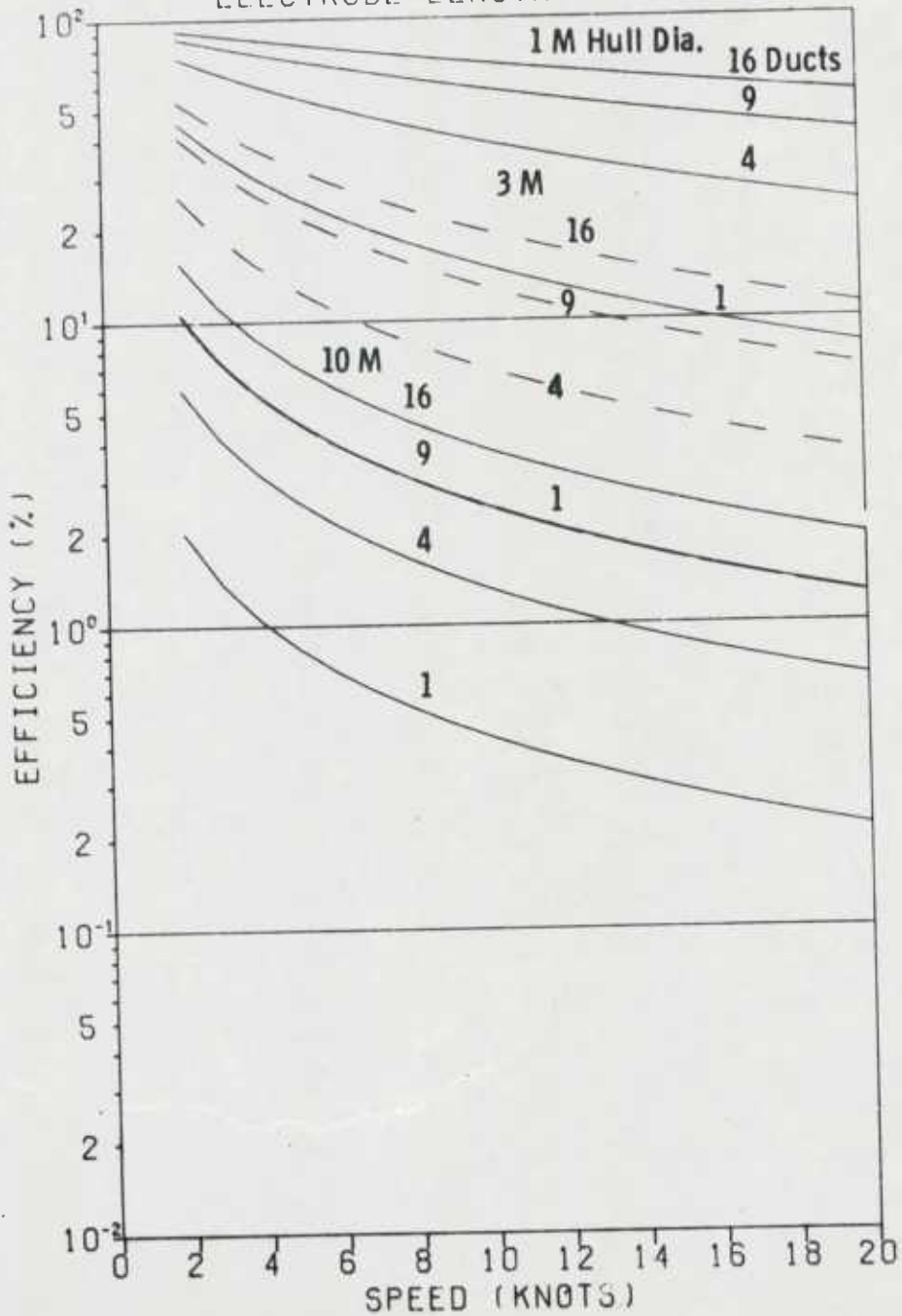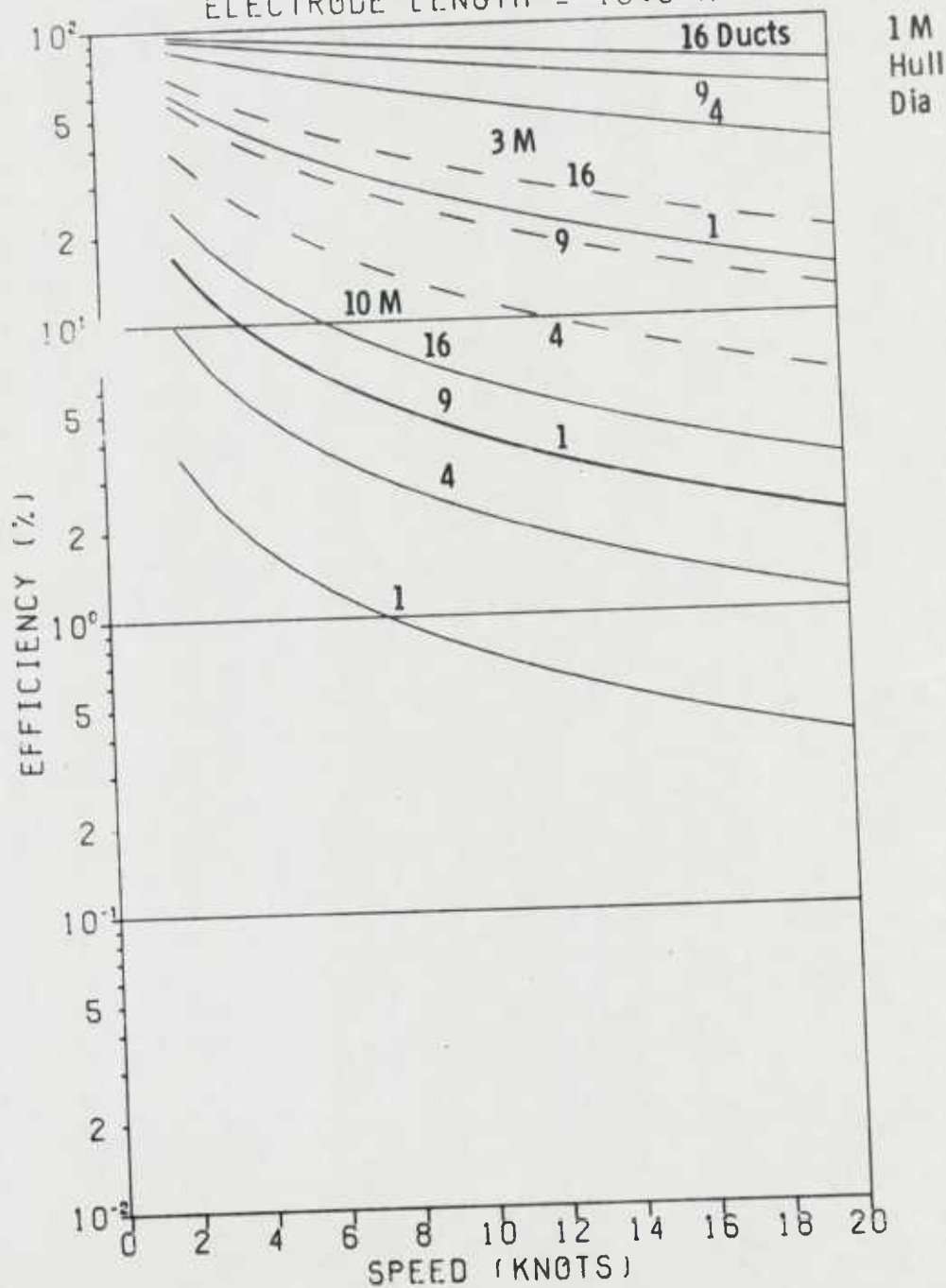EACH DUCT AREA   = .75 SQ M
ELECTRODE LENGTH = 2.0 M

FIGURE 14

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 2.0 T
EACH DUCT AREA   = 1.0 SQ M
ELECTRODE LENGTH = 2.0 M

FIGURE 15

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 2.0 T
EACH DUCT AREA   = .25 SQ M
ELECTRODE LENGTH = 10.0 M

FIGURE 16

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD = 2.0 T
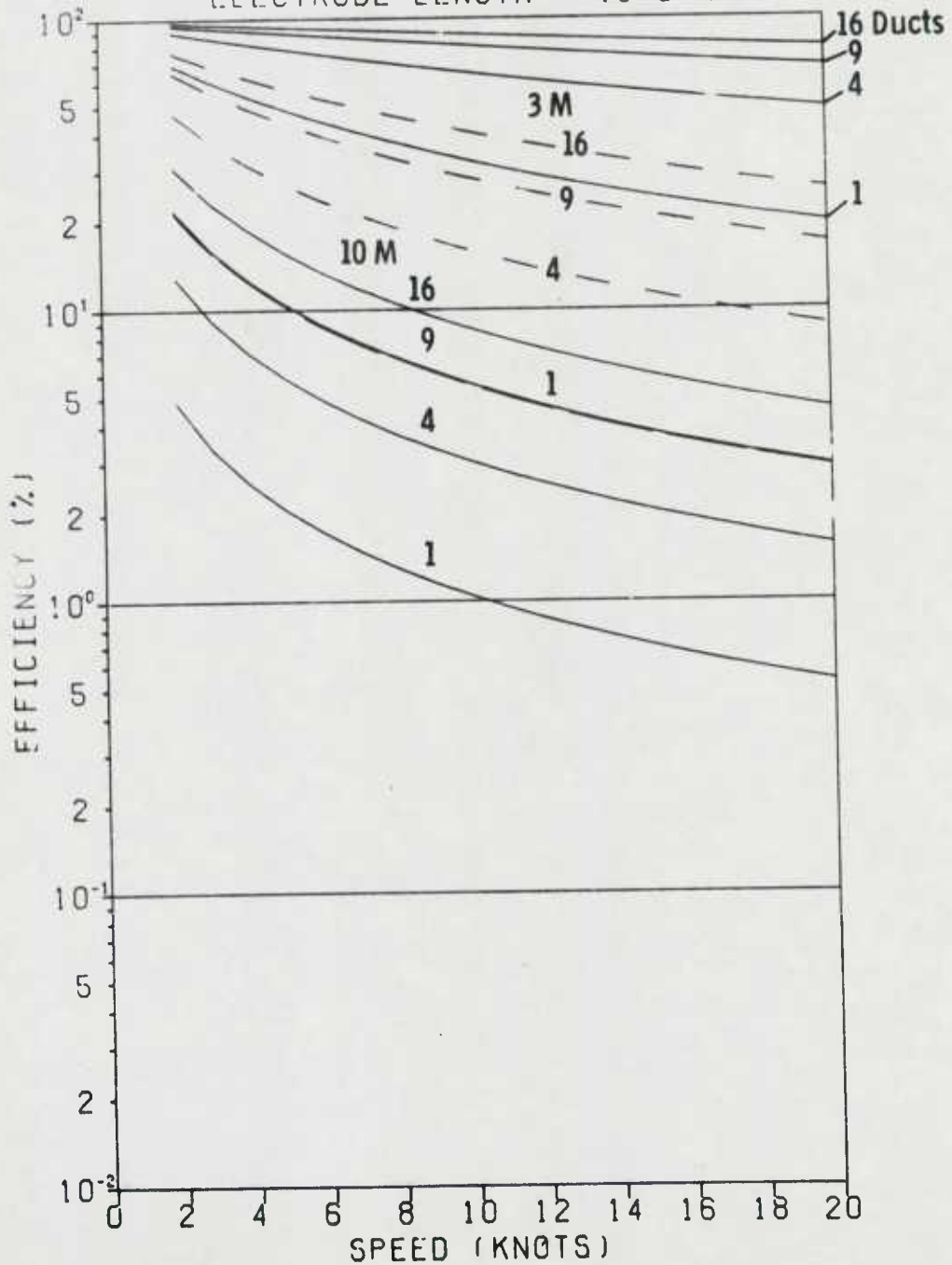EACH DUCT AREA = .50 SQ M
ELECTRODE LENGTH = 10.0 M

FIGURE 17

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 2.0 T   1 M Hull Dia.
EACH DUCT AREA   = .75 SQ M
ELECTRODE LENGTH = 10.0 M

FIGURE 18

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
MAGNETIC FIELD   = 2.0 T
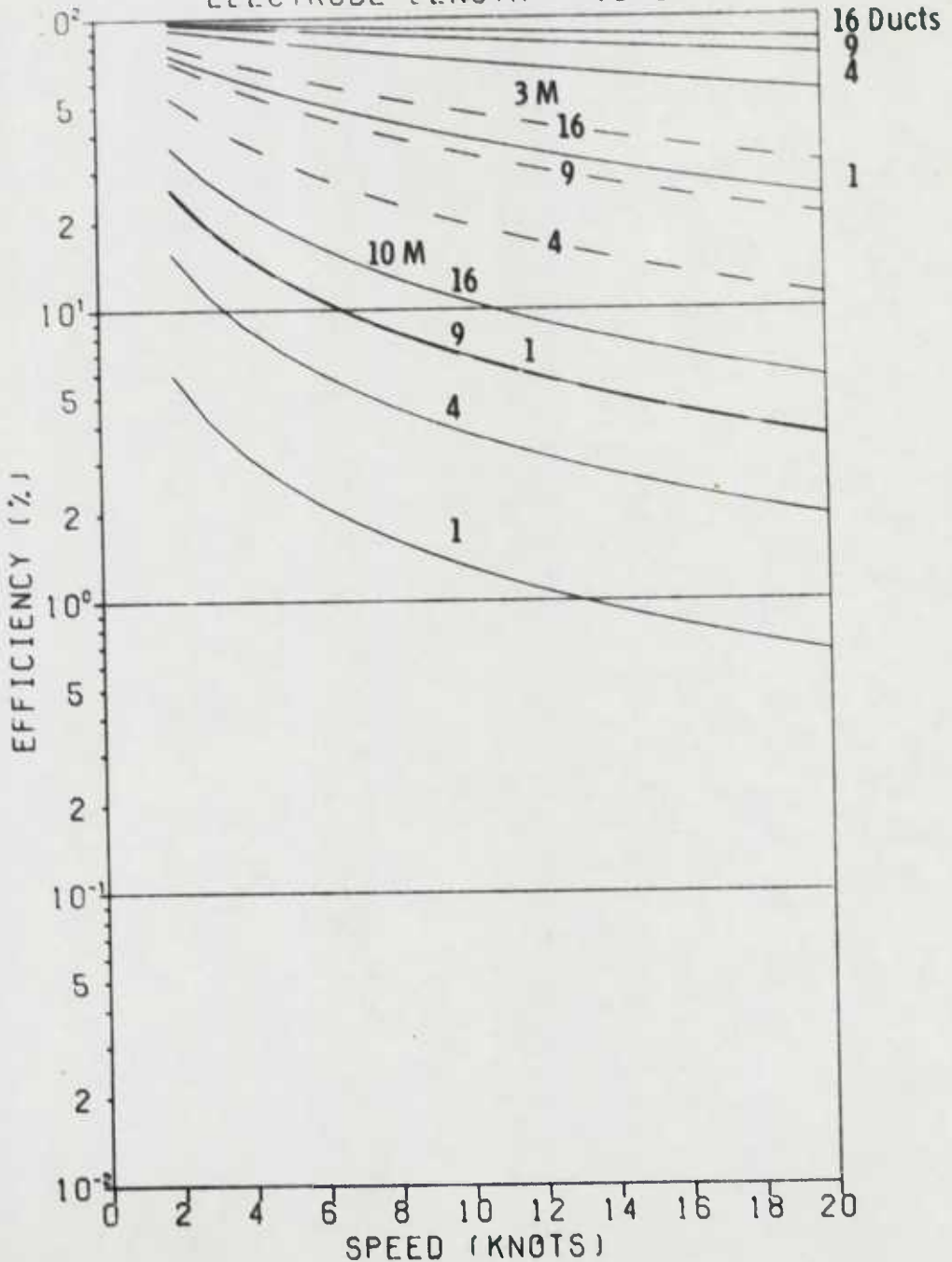EACH DUCT AREA   = 1.0 SQ M 1 M Hull Dia.
ELECTRODE LENGTH = 10.0 M

FIGURE 19

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD = 5.0 T
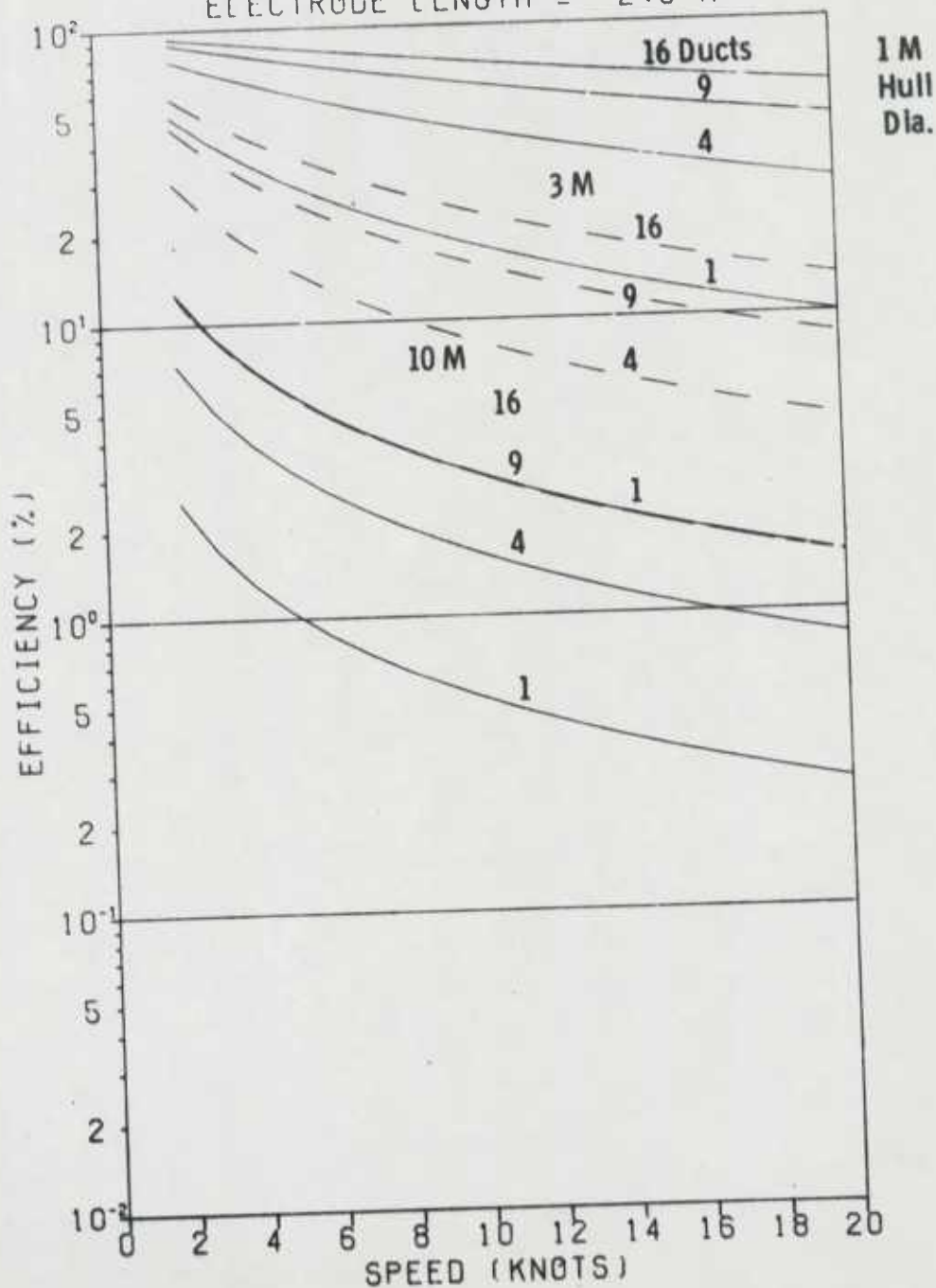EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 2.0 M

FIGURE 20

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 5.0 T
EACH DUCT AREA   = .50 SQ M
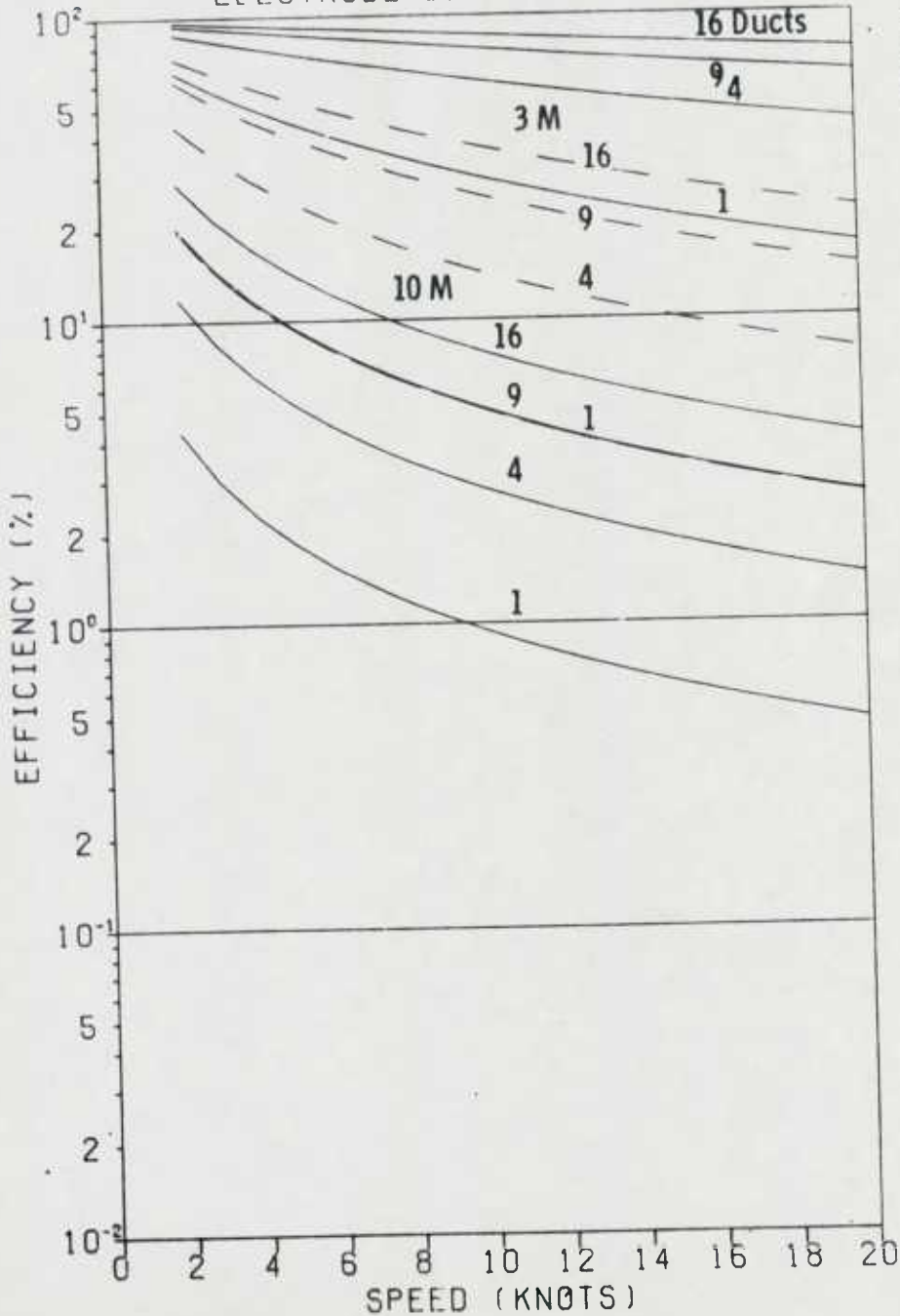ELECTRODE LENGTH = 2.0 M

FIGURE 21

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD = 5.0 T
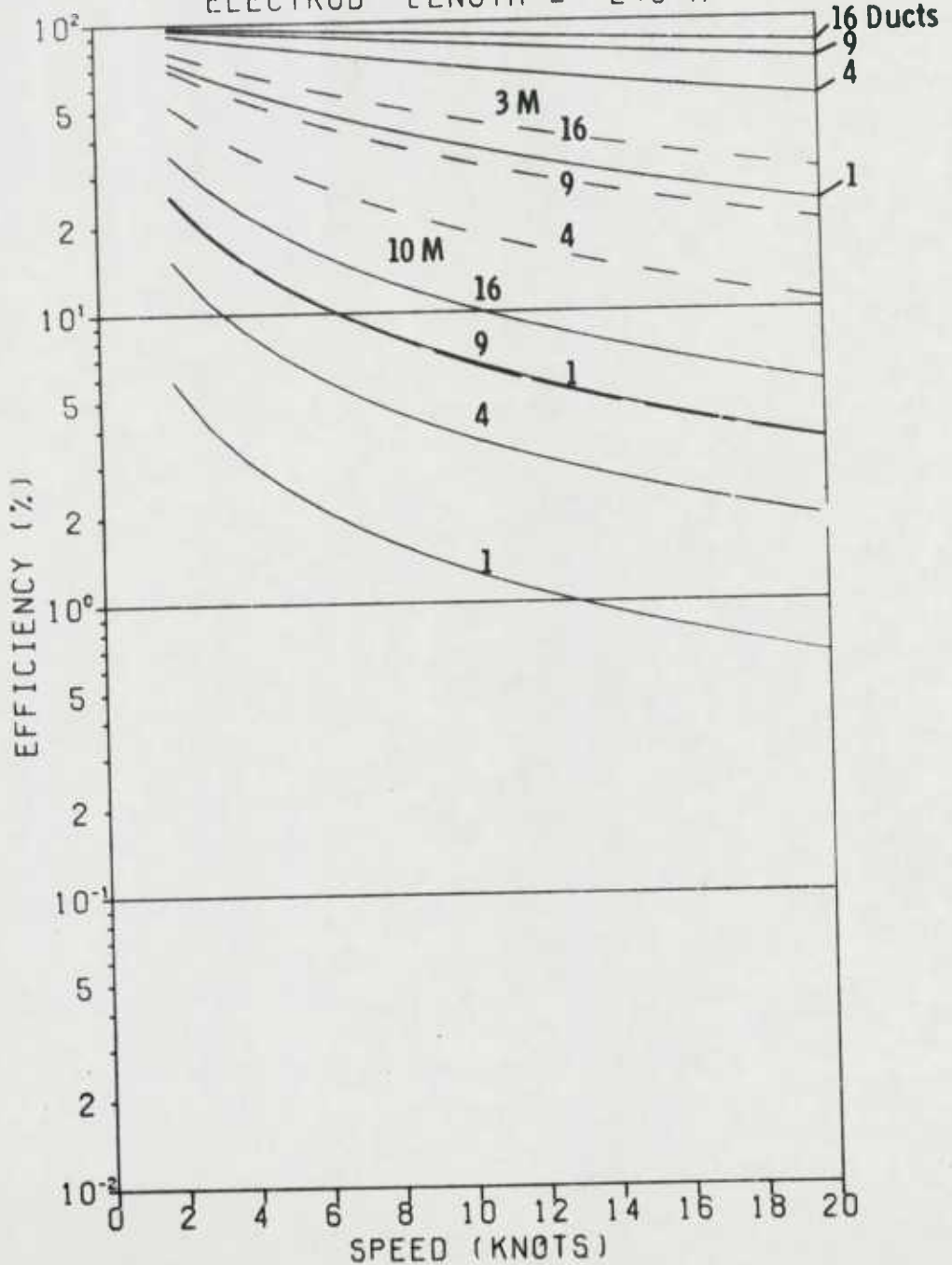EACH DUCT AREA = .75 SQ M 1 M Hull Diam.
ELECTROD LENGTH = 2.0 M

FIGURE 22

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 5.0 T
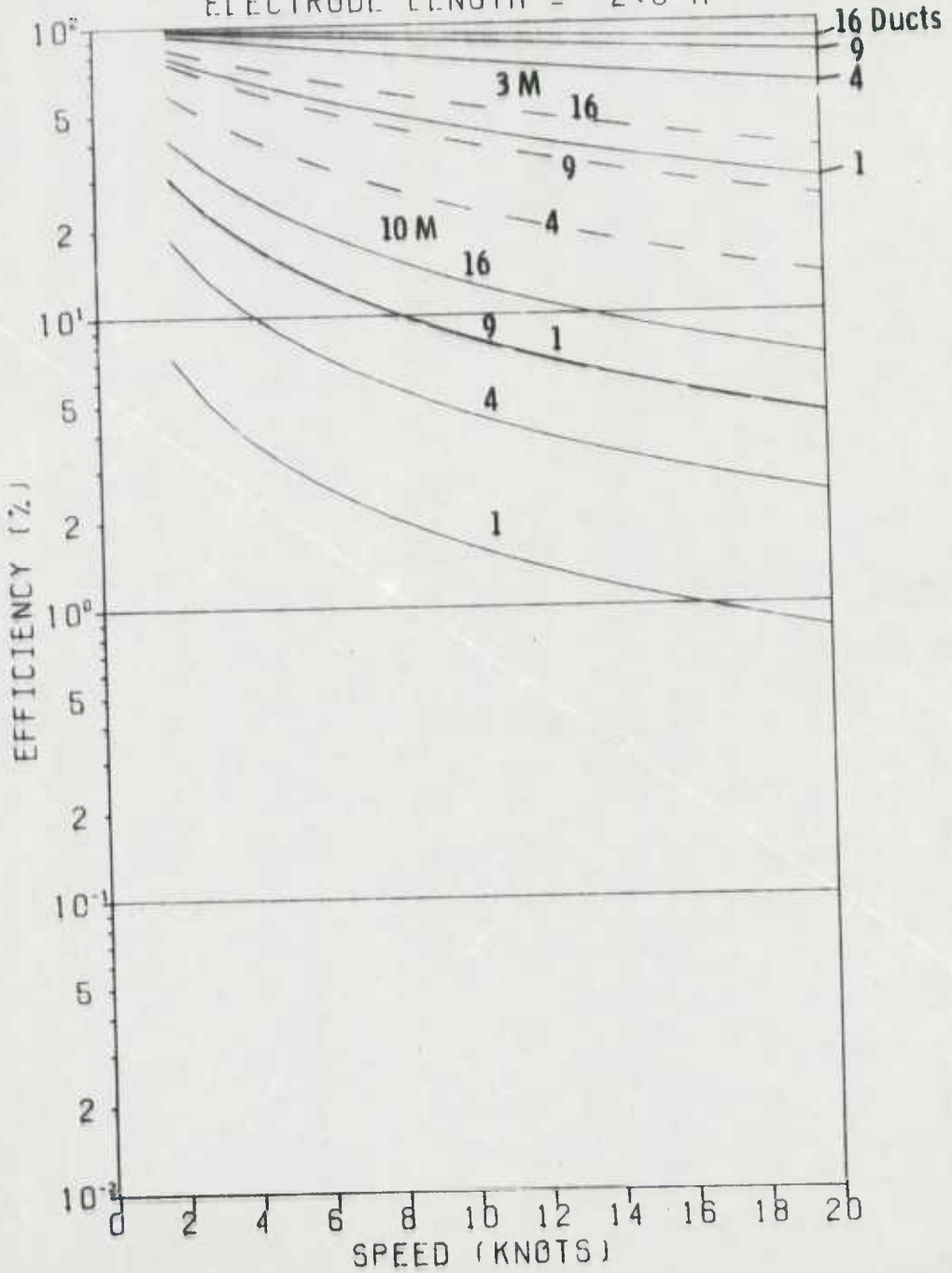EACH DUCT AREA   = 1.0 SQ M 1 M Hull Dia.
ELECTRODE LENGTH = 2.0 M

FIGURE 23

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD = 5.0 T   1 M Hull Dia.
EACH DUCT AREA = .25 SQ M
ELECTRODE LENGTH = 10.0 M
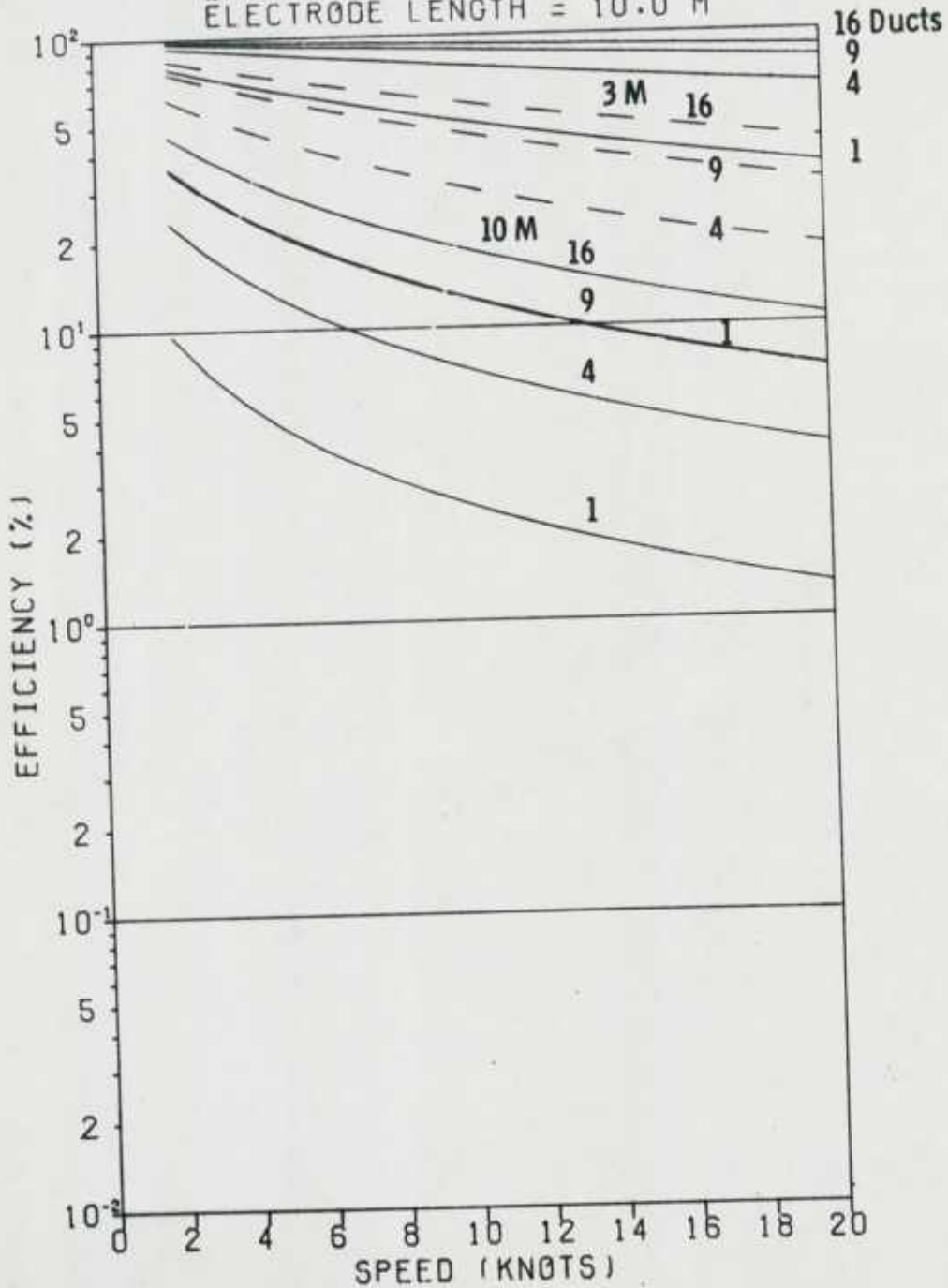
FIGURE 24

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

MAGNETIC FIELD   = 5.0 T
EACH DUCT AREA   = .50 SQ M
ELECTRODE LENGTH = 10.0 M
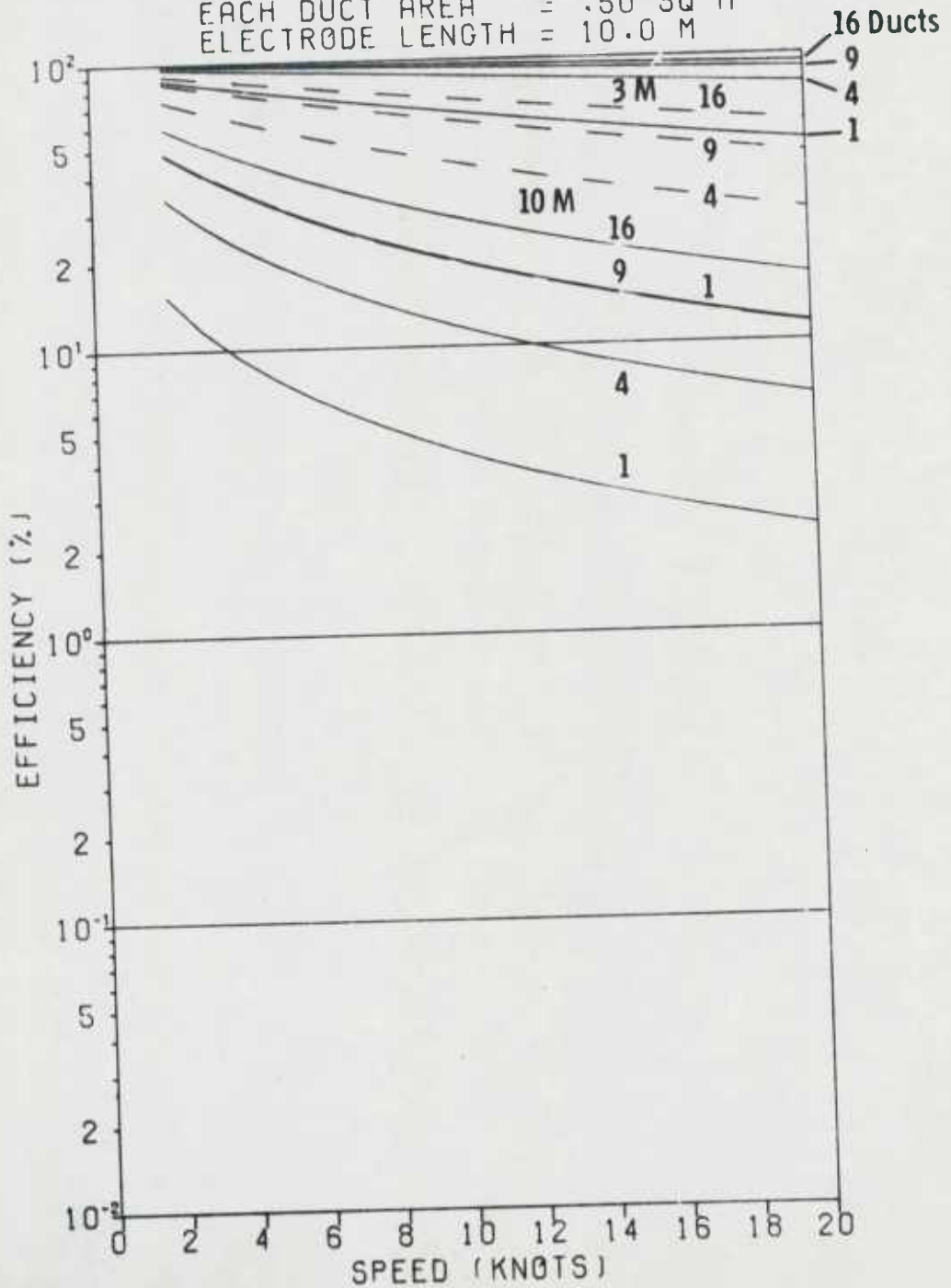
FIGURE 25

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS
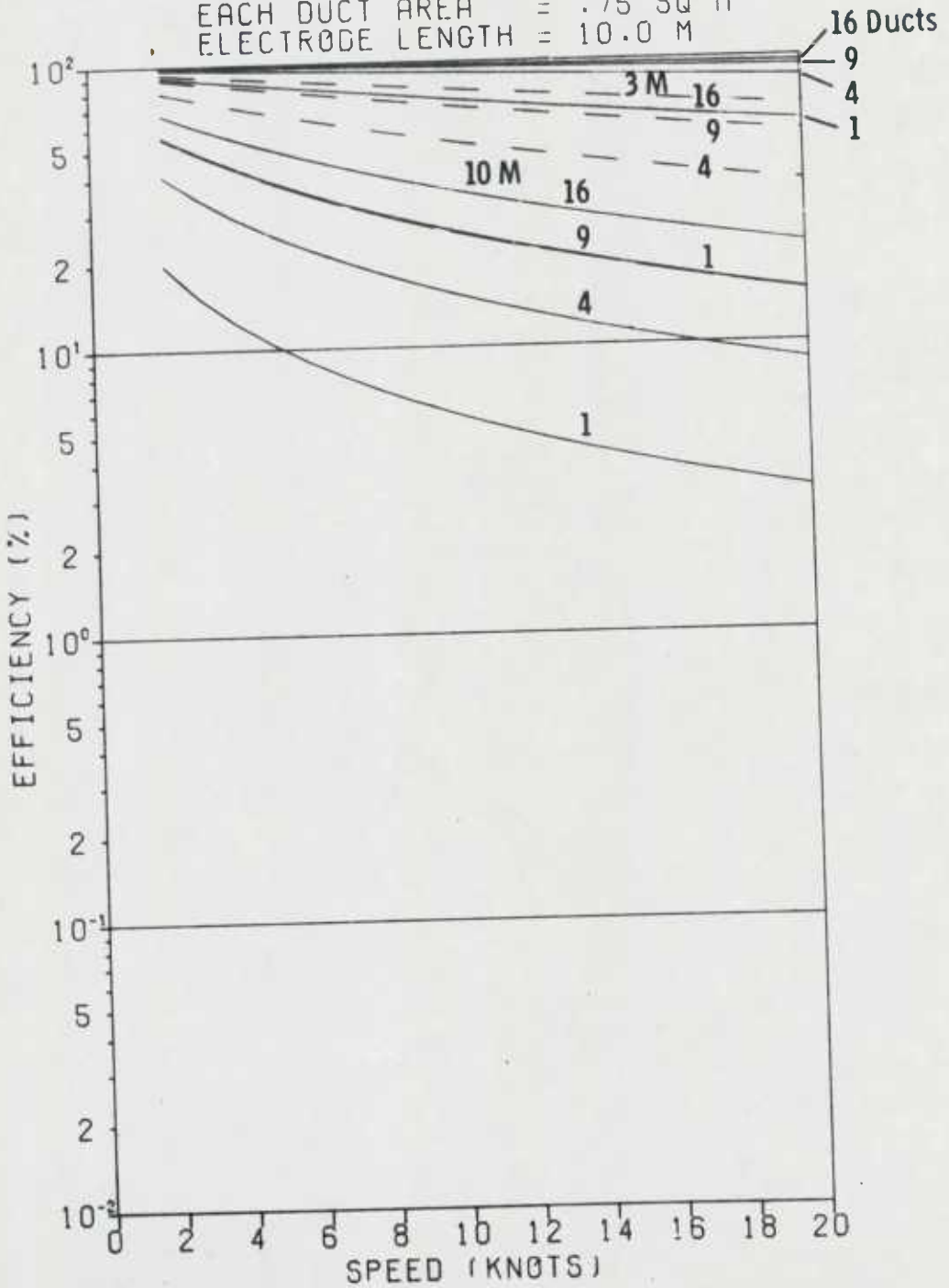
FIGURE 26

IDEAL EFFICIENCY VS SPEED
SINGLE AND MULTIPLE DUCTS

1 M Hull Dia.

MAGNETIC FIELD = 5.0 T
EACH DUCT AREA = 1.0 SQ M
ELECTRODE LENGTH = 10.0 M

16 Ducts
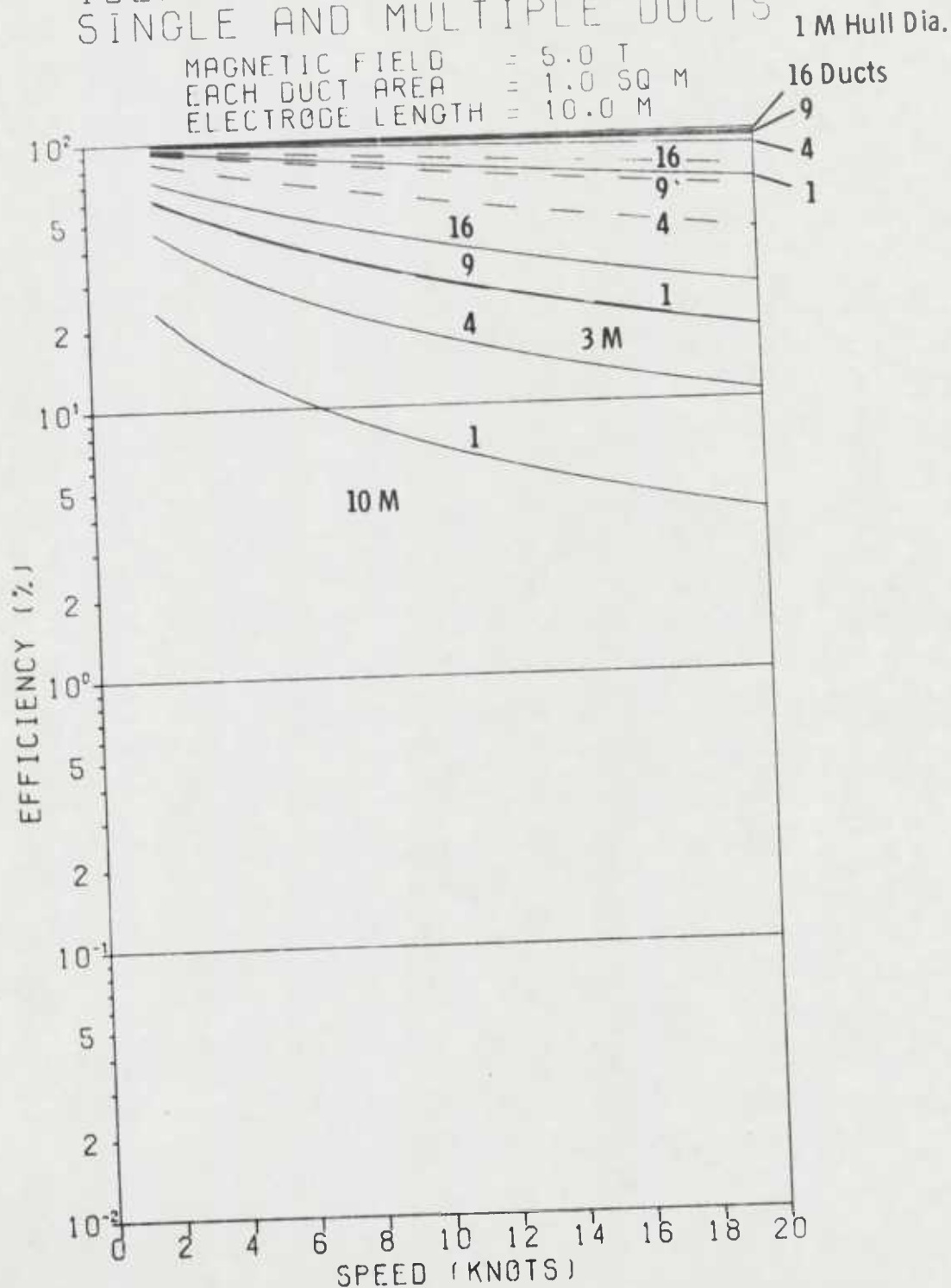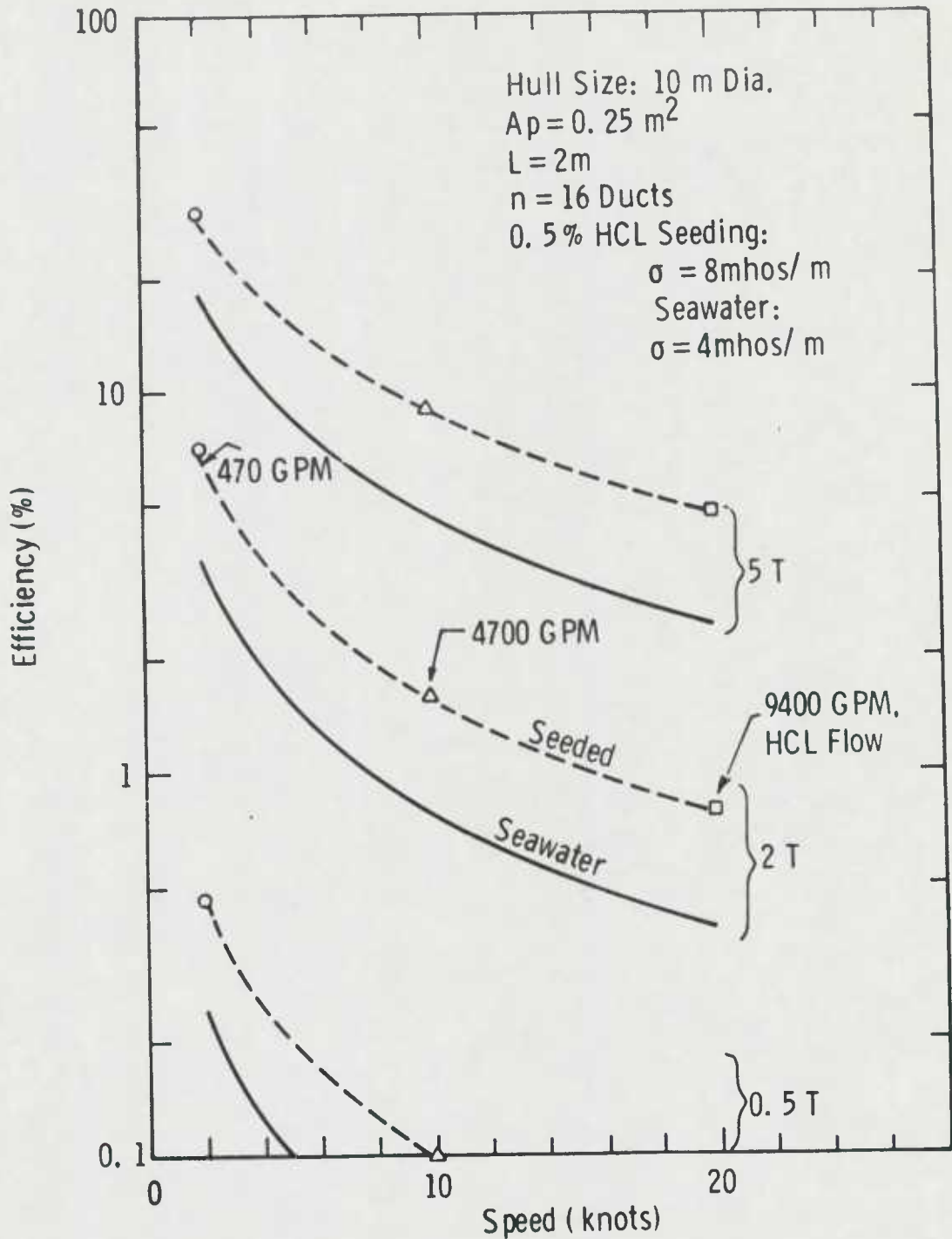
Fig.27 — Efficiency improvement via HCL seeding: full size hull
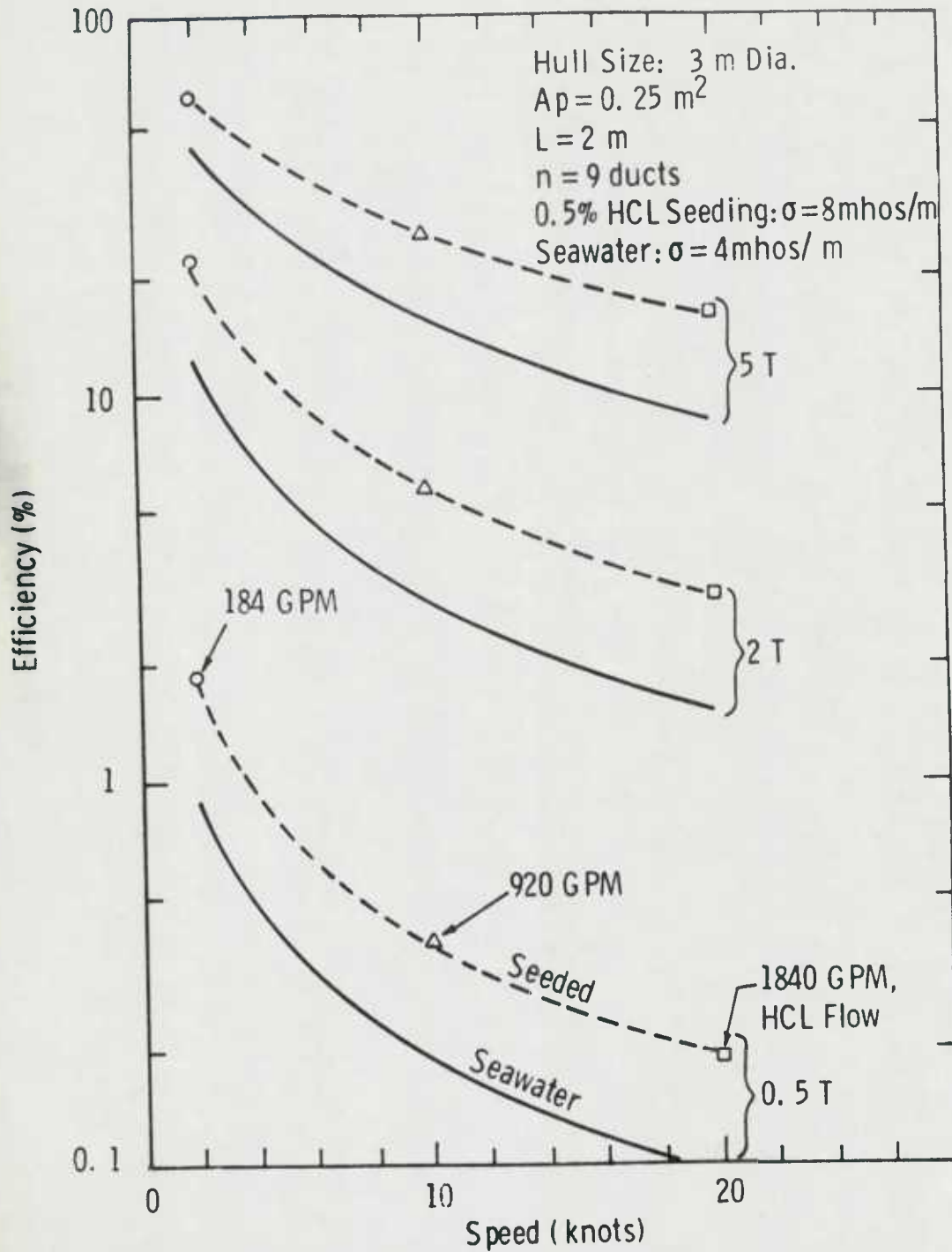
Fig. 28 — Efficiency improvement via HCL seeding: third-size hull
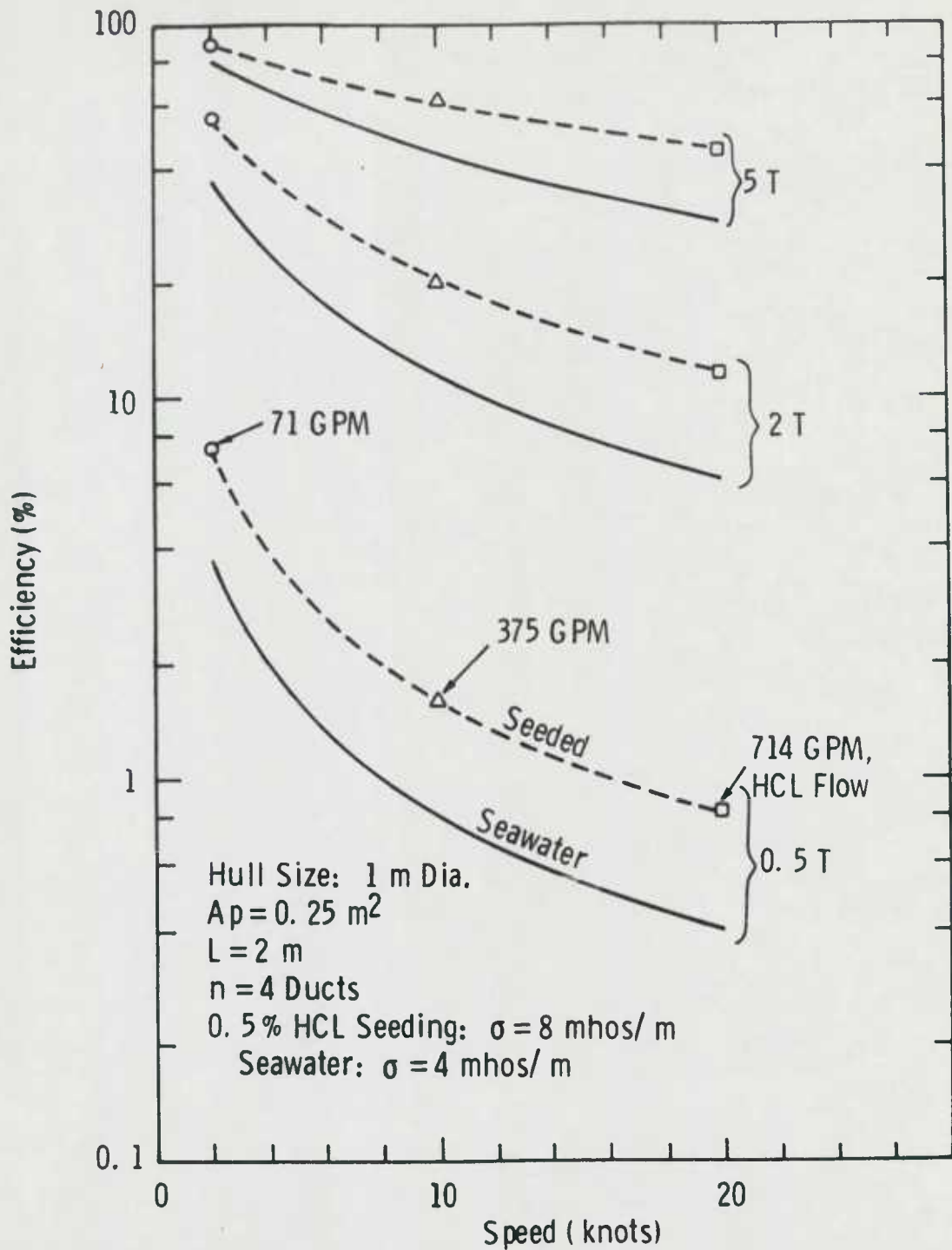
Fig. 29 – Efficiency improvement via HCL seeding: tenth-size hull
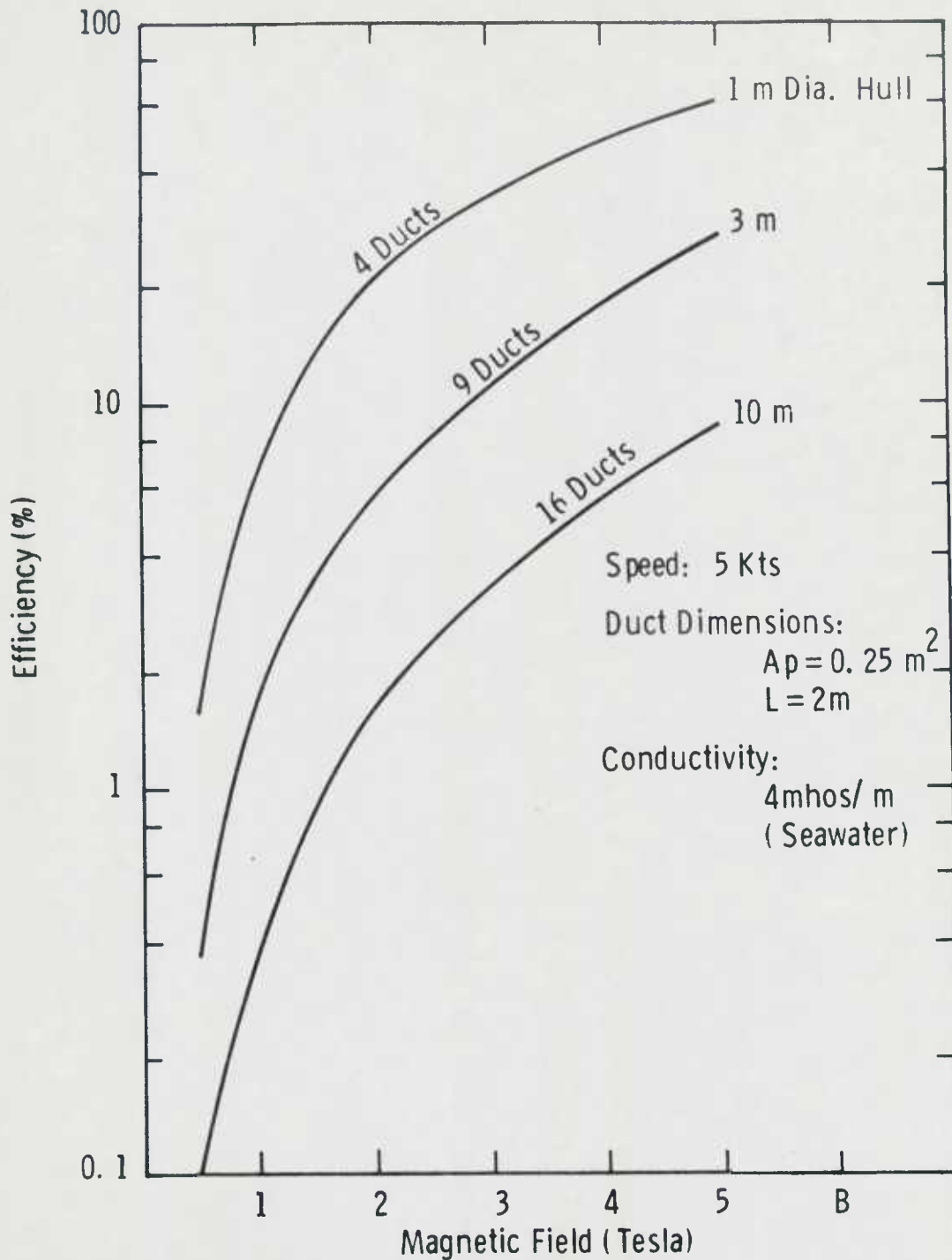
Fig. 30 — Ideal efficiency vs. magnetic field intensity for selected number of ducts & hull sizes
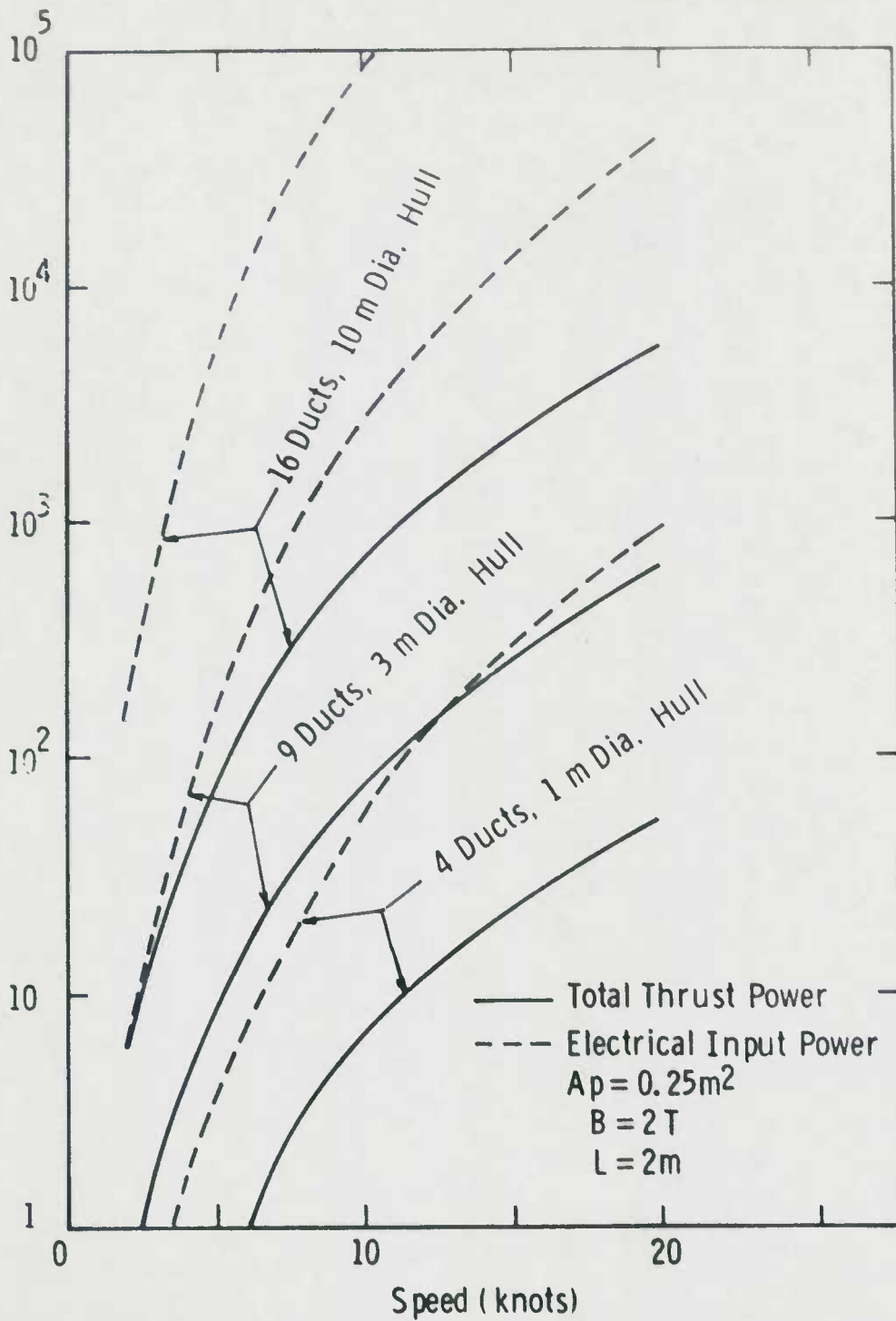
Fig. 31 — Power requirements vs. speed for selected number of ducts and hull sizes
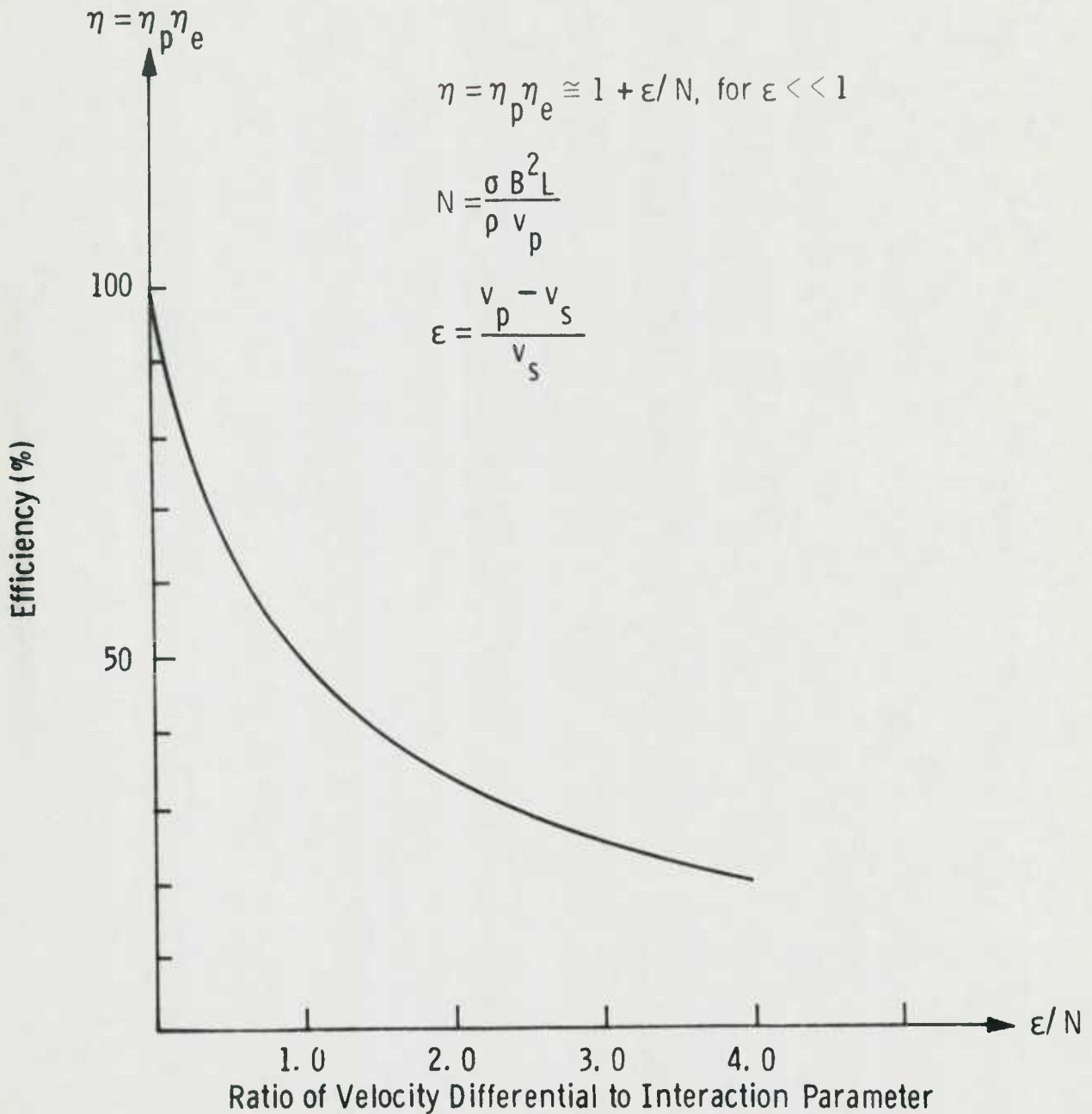
Fig. 32 — Sensitivity of ideal pump efficiency to velocity differential ÷ interaction parameter
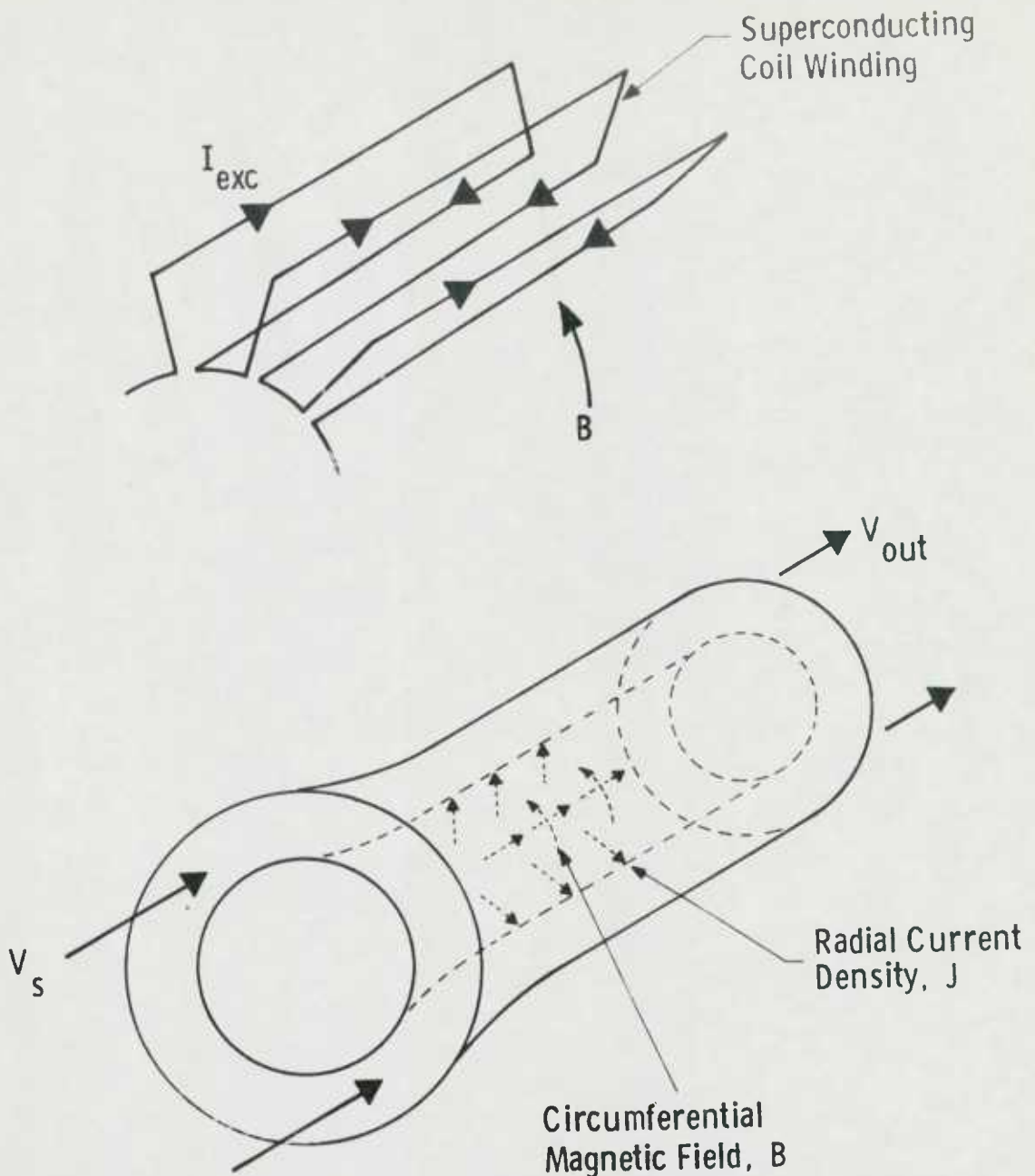
Fig. 33 — Annular pump configuration utilizing superconducting excitation. The toroidal winding establishes a high circumferential field in the pump region with low external flux leakage

APPENDIX A-1

SUBMERSIBLE HULL AND THRUSTER DRAG

The following calculations are presented to evaluate the frictional and total drag forces on a streamlined submersible of typical sizes and proportions moving at reasonable speeds, and to compare the frictional drag on a typical thruster to the total drag on the submersible. The frictional drags on the submersible and thruster are calculated by a standard velocity-squared formula that involves velocity-dependent parameters. However, the total drag characteristics used in Equation (5), Section 4.3, consist of a simple velocity-squared relationship and constant parameters. Therefore, discrepancies are likely to occur when comparing the total drag as given by Equation (5) to more accurate frictional drag calculations. The question is: "Are these discrepancies significant when compared to the total drag?"

Streamlined Hull Drag

The drag force, as used in the performance calculations in Section 4, is given by:

$$F_d = K_s v_s^2 \tag{5}$$

where K_s was determined from supplied drag vs velocity data. In order to compare this to a more accurate frictional drag, we must specify the size, shape and, hence, total submerged surface area. To do this, we assume the hull shape may be approximated by a prolate spheroid whose total surface, S, is given by:

$$S = 2 \pi a^2 + (\pi b^2/\epsilon) \cdot \ln [(1 + \epsilon)/(1 - \epsilon)] \tag{A-1.1}$$

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where 2a is the major axis, 2b is the minor axis, and ϵ is the eccentricity of the elliptical cross section:

$$\epsilon = \sqrt{1 - (b/a)^2} \qquad (A-1.2)$$

For each hull diameter, we chose a length (and eccentricity) such that the calculated frictional drag at approximately 8 knots equals the drag given by Equation (5). The three hull sizes and shapes are summarized in the following table:

Hull Dia. (m)	Length (m)	Volume (m^3)	Surface Area (m^2)	Eccentricity	Displacement (Tons)
10.0	50.0	2620	4290	.980	2890
3.3	15.5	88.4	417	.977	97.4
1.0	4.35	2.28	33.2	.975	2.46

For each combination of submersible size and speed, the Reynolds' number for the resulting flow is calculated:

$$R = v_s \, \ell/\nu \qquad (A-1.3)$$

where v_s is the free-stream speed, ℓ is the length of the submersible, and ν is the kinematic viscosity of sea water. The coefficient of friction, C_F, is interpolated from a standard table[7] that lists C_F for various values of R. (It should be noted that all of the flows are turbulent; hence, the coefficient of friction decreases for increasing Reynolds' number.) Then, the frictional drag force is calculated:

$$F_f = 1/2 \, \rho \, C_F \, S \, v_s^2 \qquad (A-1.4)$$

where ρ is the density of sea water, C_F is the frictional coefficient, S is the wetted surface area (equal to the total surface area in all cases), and v_s is the free-stream speed of the submersible. Comparison between the simple velocity-squared drag used in Section 4 and the more accurate

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frictional drag is shown in Figure A1-1. As shown in the figure, values of these two forces agree to within twenty percent for each of the three sizes, over a speed range of up to 20 knots. We conclude that the approximations for C_D and S incorporated in the constant K_s are reasonable.

Thruster Frictional Drag

The calculations for frictional drag on a thruster duct are very similar to the hull drag calculations. For the purpose of making a specific hull to thruster drag comparison, the duct length was chosen as 2 m and the cross-sectional area as .25 m^2 (this does not include inlet shroud or outlet nozzle surfaces). For these calculations the aspect ratio, α, ranges from 1 to 20. The internal surface area is exactly:

$$S = 4 L a (1 + \alpha) \qquad (A-1.5)$$

where a is the duct half width and L is the length. The coefficient of friction is taken from the same data[7] for flow over a smooth flat plate (2-sides).

Frictional force is calculated for various speeds using Equation (A-1.4). Figure A-1.2 compares duct drag obtained using this formula to hull drag taken from Figure 1. As may be seen from the figure, an aspect ratio of 1 produces the minimum drag, and the drag per duct is less than ten percent of the total drag for the small hull and less than one percent of the total for the larger hull.
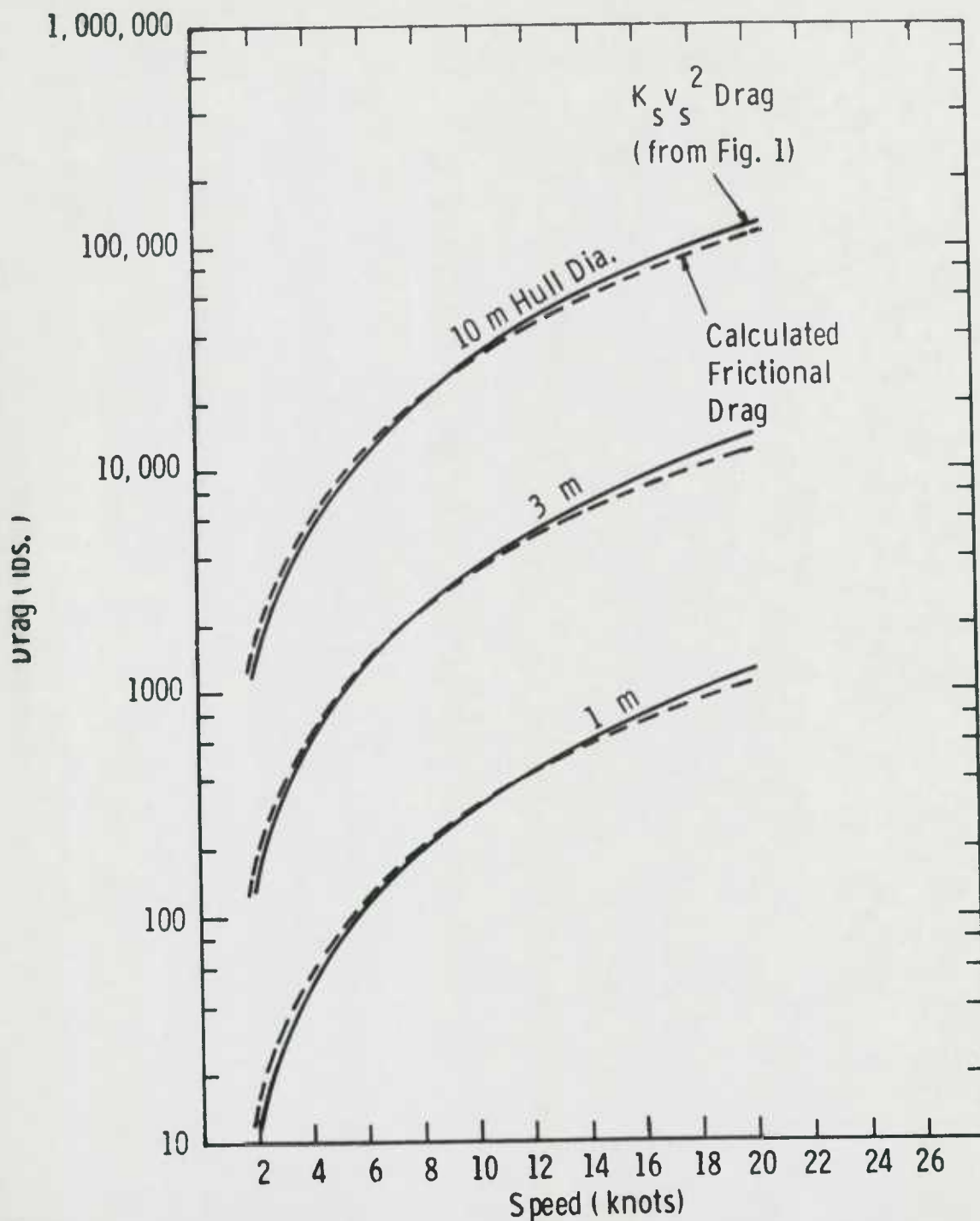
Fig. A-1. 1 — Comparison of frictional hull drag to $K_s v_s^2$ drag
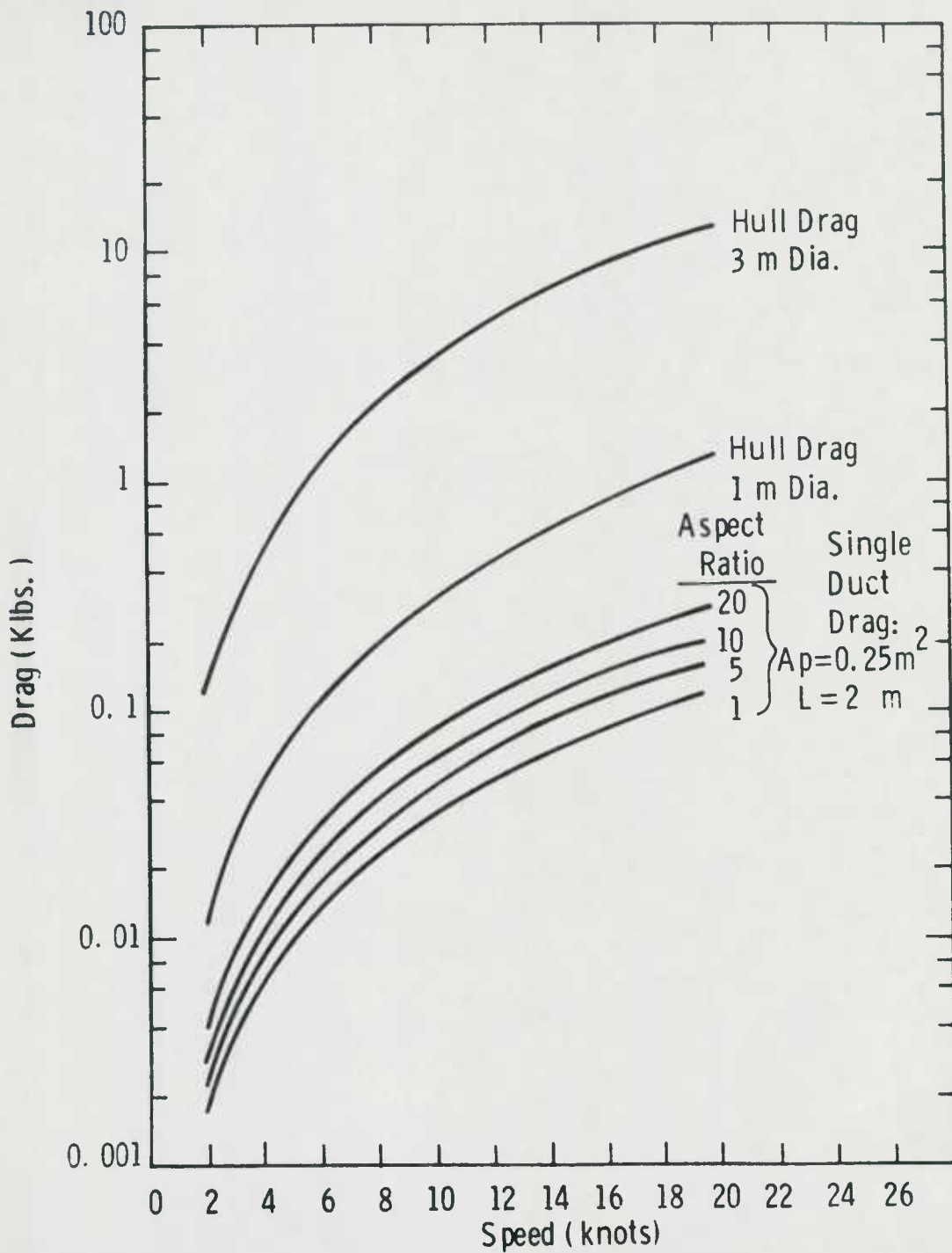
Fig. A-1. 2 — Comparison of thruster-duct drag to hull drag for selected duct size

APPENDIX A-2

ELECTROMAGNETIC COUPLING TO CONDUCTING FLUID FLOW

For fully developed incompressible flow, the vector equation of motion is given by:[8]

$$\bar{f}_b = \bar{\nabla}P - \mu_f \nabla^2 \bar{v} \qquad (A-2.1)$$

where $\bar{f}_b$ = body force

$\bar{\nabla}P$ = pressure gradient

$\mu_f \nabla^2 \bar{v}$ = viscous force density

Neglecting viscous forces and considering one-dimensional (x) motion, we have

$$\frac{\partial P}{\partial x} = (f_b)_x \qquad (A-2.2)$$

The electromagnetic body force density in a conducting fluid is given by

$$\bar{f}_b = \bar{J} \times \bar{B} \qquad (A-2.3)$$

where $\bar{J}$ = current density

$\bar{B}$ = magnetic field

If $\bar{J}$ and $\bar{B}$ are mutually perpendicular to the direction of fluid motion, as depicted in the sketch, where $\bar{J} = \hat{z} J$, $\bar{B} = -\hat{y} B$, then

$$\bar{f}_b = \hat{x} J B \qquad (A-2.4)$$

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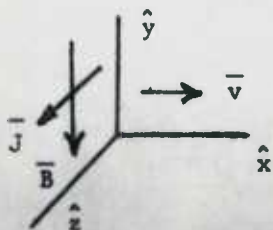
and from Equation (A-2.2), we have for the pressure gradient,

$$\frac{\partial P}{\partial x} = J B \tag{A-2.5}$$

Assuming uniform current and magnetic field intensities, over electrode length L, we may integrate this expression to obtain for the pressure rise across the pump:

$$\Delta P_p = \int_0^L J B \, dx = J B L \tag{A-2.6}$$

which is the desired expression relating pressure to current density, magnetic field, and electrode length.

Now we wish to relate current density to the difference in potential, V between the electrodes. Here we invoke Ohm's law in terms of a vector field equation for a moving medium.

$$\bar{J} = \sigma (\bar{E} + \bar{v} \times \bar{B}) \tag{A-2.7}$$

where σ = conductivity of moving medium

$\bar{B}$ = magnetic field

$\bar{v}, \bar{E}$ = velocity and electric field as seen by stationary observer

Once again neglecting fringe effects, and separating the electrodes by a distance 2b along the z axis we have:

$$\bar{E} = \hat{z} V/2b \quad , \quad \bar{v} \times \bar{B} = -\hat{z} v_p B$$

where v_p = magnitude of velocity $\bar{v}$ so that

$$J = \sigma (V/2b - v_p B) \tag{A-2.8}$$

Finally, substituting from Equation (A-2.6), we obtain Equation (14) used in Section 4.3.3:

$$\Delta P_p = \sigma B L (V/2b - v_p B) \tag{14}$$

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APPENDIX A-3

A3.1 Fortran Source Listing:

        Calculations of ideal efficiency, voltage and current

        as function of:

                magnetic field

                area

                number of ducts

                hull size

                electrode length

A3.2 Sample Printout

A3.1 Source

```
@RUN, /RNPT SUBCAL.09B23NSFMO01.EMSUB.1.25

@DATA.IL          TPFS.
DATA 7R1 SL74T9 08/06/79 11:29:54 (->0)
                      DATA LINE /6/
    1.                REAL KO.K1.KS
    2.                DIMENSION EFFARY(3,4,20),XV(21),YV(21)
    3.          C
    4.          C         EFFARY, XV.YV ALL USED FOR CALCOMP OUTPUT
    5.          C         SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
    6.          C
    7.          C         VS IN METERS/SEC
    8.          C
    9.          C         AO IN SQ METERS
   10.          C
   11.          C         KO IN NEWTONS/(M/S)**2
   12.          C           LBS THRUST/KNOT**2 = KS*ALPHA
   13.          C
   14.          C         SPEED IN KNOTS     =  VS*BETA
   15.          C
   16.          C         RHO = DENSITY,KG/M**3
   17.          C
   18.          C         SIGMA = CONDUCTIVITY,MHOS
   19.          C
   20.          C
   21.                KREF = 5000.
   22.                AO =.25
   23.                SIGMA = 4.
   24.                RHO = 1.E3
   25.                VPRIME = 1.
   26.          C         VPRIME = KNOTS
   27.                ALPHA=((12.57)**2)*6973.) /((31.E3)*25.)
   28.                BETA = 5./2.57
   29.                VSO=VPRIME/BETA
   30.          C     VSO IN M/S
   31.          C
   32.          C
   33.          C         JMAX     .........# OF DUCTS
   34.          C           LMAX...:..          VELOCITY
   35.          C             MMAX...........  BFIELD
   36.          C           KMAX.....DUCT AREA
   37.          C      NMAX.... ELECRRODE LENGTH
   38.          C
   39.                JMAX = 4
   40.                LMAX = 19
   41.                KMAX=4
   42.                MMAX=3
   43.                NMAX=2
   44.                DO 100 M= 1.MMAX
   45.                   IF(M.EQ.1) B = .5
   46.                   IF(M.EQ.2) B = 2.
   47.                   IF(M.GE.3) B = 5.
   48.          C
   49.          C         DIST IN METERS
   50.                DO 200 N = 1.NMAX
   51.                   IF(N.EQ.1) DIST =2.
   52.                   IF(N.GE.2) DIST=10.
   53.          C
   54.          C
   55.          C
   56.          C
   57.                DO 300 K= 1.KMAX
   58.                   RK =K
   59.                   A = AO*RK
   60.                   SIDE =SQRT(A)
   61.                DO 400 JSIZ = 1,3
   62.                   IF(JSIZ.EQ.1) KO = KREF
   63.                   IF(JSIZ.EQ.2) KO = KREF/9.
   64.                   IF(JSIZ.EQ.3) KO = KREF/100.
   65.                DO 400 J=1.JMAX
   66.                   RJ=J
   67.                   KS= KO/(RJ*RJ)
   68.                   ALPHAK =ALPHA*KS
   69.                   K1 = (1.+SQRT(1.+4.*KS/(RHO*A)))/2.
   70.                   F = K1*K1 - 1.
   71.
```

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```
72:          WRITE(LINE,101)  KS,ALPHAK,A,K1
73:    101   FORMAT(1H ,/,'KS = ',E9.3,' NT/(M/S)2 = ',E9.3,' LB/(KT)2  ...',
74:         1 'AREA = ',F9.3,' SQ METERS',10...',K1 = ',
75:         2F9.3,'= RATIO VIN/VS',/)
76:          WRITE(LINE,103)
77:    103   FORMAT(1H ,5X,'VS(M/S)   VS(KTS)   B(TESLA)  L(METERS) CUR(KAMP)
78:         1 VOLTS    PRESS(PSI) TH(KNTS)  TH(KLBS)  EFFICIENCY(%)',/)
79:    C
80:          DO 500 L = 1,LMAX
81:    C           COMPUTE  EVFICIENCY FOR VS= 2, 20 KTS
82:                RL = L+1
83:                VS= VSO+RL
84:                BETAVS = BETA+VS
85:
86:    C     CONVERT  VS TO KNOTS
87:    C
88:    C
89:    C
90:    C
91:                VZRO =(2.+SIGMA+(B++2)+DIST)/RHO
92:                EFF  =(2.+KS/(A+RHO))/(F+(VS+F/VZRU +K1))
93:    C
94:    C     HERE WE ARE ASSUMING  ASPECT RATIO = 1 ,THEN
95:    C
96:    C           AREA = SIDE++2
97:    C
98:    C
99:                CUR = SIDE+RHO+(VS++2)+((K1++2)-1.)/B
100:   C     USE SIDE/2 IN CURRENT EXPRESSION
101:                CUR=CUR/2.
102:                TH  = KS+VS++2
103:                THPDS =TH+(4973./31.E3)
104:                VOLT= TH+VS/(CUR+EFF)
105:                DELP = RHO+VS+VS+(K1+K1-1.)/2.
106:   C     CONVERT DELP FROM NT/SQM TO PSI
107:                PSI = DELP+1.45E-4
108:   C
109:          WRITE(LINE,501) VS,BETAVS,B,DIST,CUR,VOLT,
110:         1PSI,TH,THPDS,EFF
111:   501   FORMAT(1H ,3X,4(F9.3,1X),-3PF9.3,1X,0PF9.3,1X,F10.2,1X
112:         12(-3PF9.3,1X),2PF9.3)
113:   500            CONTINUE
114:   400        CONTINUE
115:   C
116:   C
117:          WRITE(LINE,301)
118:   301   FORMAT(1H ,/)
119:   300        CONTINUE
120:   200      CONTINUE
121:   100    CONTINUE
122:          STOP
123:          END
124:
```

END DATA. ERRORS: NONE. TIME: 0.692 SEC. IMAGE COUNT: 124

BFIN

a = 4.0 mhos/m

ELECTROMAGNETIC THRUST PROPULSION

K_S =   .500+04 NT/(M/S)2   =   .297+03 LB/(KT)2 ***AREA =   1.000 SQ METERS***K1 =   2.791= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	7.177	898.550	.52	5.284	1.189	.084
1.542	3.000	.500	2.000	11.148	2020.662	1.17	11.889	2.674	.056
2.056	4.000	.500	2.000	15.708	3591.331	2.08	21.136	4.754	.042
2.570	5.000	.500	2.000	20.856	5610.559	3.25	33.024	7.428	.034
3.084	6.000	.500	2.000	26.592	8078.344	4.68	47.555	10.697	.028
3.598	7.000	.500	2.000	32.917	10994.686	6.37	64.748	14.560	.024
4.112	8.000	.500	2.000	39.831	14359.687	8.33	84.543	19.017	.021
4.626	9.000	.500	2.000	47.333	18173.045	10.54	106.999	24.068	.019
5.140	10.000	.500	2.000	55.423	22435.061	13.01	132.098	29.714	.017
5.654	11.000	.500	2.000	64.102	27145.635	15.74	159.839	35.953	.015
6.168	12.000	.500	2.000	73.369	32304.766	18.73	190.221	42.787	.014
6.682	13.000	.500	2.000	83.225	37912.455	21.98	223.246	50.216	.013
7.196	14.000	.500	2.000	93.669	43968.703	25.50	258.912	58.239	.012
7.710	15.000	.500	2.000	104.702	50473.506	29.27	297.220	66.855	.011
8.224	16.000	.500	2.000	116.323	57426.870	33.30	338.171	76.067	.010
8.738	17.000	.500	2.000	128.531	64828.790	37.59	381.763	85.872	.009
9.252	18.000	.500	2.000	141.331	72679.267	42.15	427.997	96.272	.009
9.766	19.000	.500	2.000	154.717	80978.303	46.96	476.874	107.266	.008
10.280	20.000	.500	2.000	168.692	89725.896	52.03	528.392	118.854	.008

K_S =   .125+04 NT/(M/S)2   =   .743+02 LB/(KT)2 ***AREA =   1.000 SQ METERS***K1 =   1.725= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	2.087	261.746	.15	1.321	.297	.249
1.542	3.000	.500	2.000	4.195	588.264	.34	2.972	.669	.166
2.056	4.000	.500	2.000	6.348	1045.212	.61	5.284	1.189	.125
2.570	5.000	.500	2.000	8.563	1632.590	.95	8.256	1.857	.100
3.084	6.000	.500	2.000	10.854	2350.398	1.36	11.889	2.674	.071
3.598	7.000	.500	2.000	13.190	3198.636	1.85	16.182	3.640	.062
4.112	8.000	.500	2.000	15.671	4177.303	2.42	21.136	4.754	.055
4.626	9.000	.500	2.000	18.128	5286.401	3.06	26.750	6.017	.050
5.140	10.000	.500	2.000	20.828	6526.929	3.78	33.024	7.428	.045
5.654	11.000	.500	2.000	23.573	7895.886	4.58	39.960	8.988	.042
6.168	12.000	.500	2.000	26.478	9396.271	5.45	47.555	10.697	.038
6.682	13.000	.500	2.000	29.171	11027.090	6.39	55.811	12.554	.036
7.196	14.000	.500	2.000	32.357	12788.337	7.41	64.728	14.560	.033
7.710	15.000	.500	2.000	35.502	14680.012	8.51	74.305	16.714	.031
8.224	16.000	.500	2.000	38.777	16854.615	9.68	84.543	19.017	.029
8.738	17.000	.500	2.000	41.930	21113.625	10.93	95.441	21.468	.028
9.252	18.000	.500	2.000	45.341	23551.022	12.26	106.999	24.068	.026
9.766	19.000	.500	2.000	48.888	26044.248	13.65	119.218	26.816	.025
10.280	20.000	.500	2.000			15.13	132.098	29.714	.023

K_S =   .556+03 NT/(M/S)2   =   .330+02 LB/(KT)2 ***AREA =   1.000 SQ METERS***K1 =   1.398= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	1.007	126.619	.07	.587	.132	.473
1.542	3.000	.500	2.000	2.266	284.353	.16	1.321	.297	.316
2.056	4.000	.500	2.000	4.029	505.035	.29	2.348	.528	.237
2.570	5.000	.500	2.000	6.295	788.673	.44	3.669	.825	.190
3.084	6.000	.500	2.000	9.065	1135.258	.64	5.284	1.189	.158
3.598	7.000	.500	2.000	12.338	1544.793	.89	7.192	1.618	.136
4.112	8.000	.500	2.000	16.115	2017.279	1.17	9.394	2.113	.119
4.626	9.000	.500	2.000	20.396	2552.715	1.48	11.889	2.674	.106
5.140	10.000	.500	2.000	25.180	3151.100	1.83	14.678	3.302	.095
5.654	11.000	.500	2.000	30.468	3812.436	2.21	17.760	3.995	.086
6.168	12.000	.500	2.000	36.260	4536.723	2.63	21.136	4.754	.079
6.682	13.000	.500	2.000	42.554	5323.959	3.09	24.805	5.580	.073
7.196	14.000	.500	2.000	49.353	6174.146	3.58	28.768	6.471	.068
7.710	15.000	.500	2.000	56.655	7087.282	4.11	33.024	7.428	.063
8.224	16.000	.500	2.000	64.461	8063.369	4.67	37.575	8.452	.059
8.738	17.000	.500	2.000	72.770	9102.406	5.28	42.418	9.541	.056
9.252	18.000	.500	2.000	81.583	10204.393	5.91	47.555	10.697	.053
9.766	19.000	.500	2.000	90.900	11369.331	6.59	52.986	11.918	.050
10.280	20.000	.500	2.000	100.720	12597.218	7.30	58.710	13.206	.048

K_S =   .312+03 NT/(M/S)2   =   .186+02 LB/(KT)2 ***AREA =   1.000 SQ METERS***K1 =   1.250= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)

Full size
1 Duct

4 Ducts

9 Ducts

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16 Ducts

K_S = .556+03 NT/(M/S)2 = .330+02 LB/(KT)2 ***AREA =   1.000 SQ METERS***K1 =   1.398= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)

THIRD SIZE
1 DUCT

K_S = .139+03 NT/(M/S)2 = .825+01 LB/(KT)2 ***AREA =   1.000 SQ METERS***K1 =   1.124= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.277	35.253	.02	.147	.033	1.543
1.042	3.000	.500	2.000	.624	78.886	.05	.330	.074	1.034
1.056	4.000	.500	2.000	1.110	139.857	.08	.587	.132	.778
1.570	5.000	.500	2.000	1.734	218.166	.12	.917	.206	.623
2.497	6.000	.500	2.000	2.497	313.812	.18	1.321	.297	.520
3.398	7.000	.500	2.000	3.398	427.796	.25	1.798	.404	.446
4.112	8.000	.500	2.000	4.438	557.118	.32	2.348	.528	.391
4.626	9.000	.500	2.000	6.935	704.778	.41	2.972	.669	.347
5.140	10.000	.500	2.000	6.391	869.776	.50	4.440	.825	.313
5.654	11.000	.500	2.000	9.987	1052.111	.61	5.284	.999	.284
6.168	12.000	.500	2.000	11.720	1251.784	.72	6.201	1.189	.261
6.682	13.000	.500	2.000	13.593	1468.795	.85	7.192	1.395	.241
7.196	14.000	.500	2.000	15.604	1703.143	.99	8.256	1.618	.224
7.710	15.000	.500	2.000	17.754	1954.829	1.13	9.394	1.857	.209
8.224	16.000	.500	2.000	20.042	2223.853	1.29	10.605	2.113	.196
8.738	17.000	.500	2.000	22.470	2510.215	1.45	11.889	2.385	.184
9.252	18.000	.500	2.000	25.036	2813.915	1.63	13.246	2.674	.174
9.766	19.000	.500	2.000	27.740	3134.953	1.82	14.675	2.980	.165
10.280	20.000	.500	2.000		3473.327	2.01		3.302	.157

4 DUCTS

K_S = .617+02 NT/(M/S)2 = .367+01 LB/(KT)2 ***AREA =   1.000 SQ METERS***K1 =   1.058= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	2.000	.127	16.403	.01	.065	.015	3.222
1.542	3.000	.500	2.000	.285	36.499	.02	.147	.033	2.172
2.056	4.000	.500	2.000	.507	64.524	.04	.261	.059	1.638
2.570	5.000	.500	2.000	.793	100.479	.06	.408	.092	1.315

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VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)	
3.084	6.000	.500	2.000	1.142	144.363	.08	.587	.132	1.098	9 Ducts
3.598	7.000	.500	2.000	1.554	196.177	.11	.799	.180	.943	
4.112	8.000	.500	2.000	2.030	255.920	.15	1.044	.235	.826	
4.626	9.000	.500	2.000	2.569	323.593	.19	1.321	.297	.735	
5.140	10.000	.500	2.000	3.172	399.195	.23	1.561	.367	.682	
5.654	11.000	.500	2.000	3.838	482.727	.28	1.973	.444	.602	
6.168	12.000	.500	2.000	4.567	574.188	.33	2.348	.528	.552	
6.682	13.000	.500	2.000	5.360	673.779	.39	2.750	.620	.510	
7.196	14.000	.500	2.000	6.137	780.899	.45	3.190	.719	.474	
7.710	15.000	.500	2.000	8.120	896.149	.52	3.669	.825	.442	
8.224	16.000	.500	2.000	9.137	1019.328	.59	4.175	.939	.415	
8.738	17.000	.500	2.000		1150.437	.66	4.713	1.060	.391	
9.252	18.000	.500	2.000	10.277	1289.475	.75	5.284	1.189	.369	
9.766	19.000	.500	2.000	11.450	1436.443	.83	5.887	1.324	.350	
10.280	20.000	.500	2.000	12.687	1591.340	.92	6.523	1.467	.332	

K_s = .347+02 NT/(M/S)2 = .206+01 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.034= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)	
1.028	2.000	.500	2.000	.072	9.556	.01	.037	.008	5.468	
1.542	3.000	.500	2.000	.162	21.102	.01	.083	.019	3.714	
2.056	4.000	.500	2.000	.286	37.100	.02	.147	.033	2.812	
2.570	5.000	.500	2.000	.431	57.711	.03	.229	.052	2.263	
3.084	6.000	.500	2.000	.650	82.013	.04	.330	.074	1.893	
3.598	7.000	.500	2.000	.884	112.408	.06	.450	.101	1.627	16 Ducts
4.112	8.000	.500	2.000	1.155	146.515	.08	.587	.132	1.426	
4.626	9.000	.500	2.000	1.462	185.135	.11	.743	.167	1.270	
5.140	10.000	.500	2.000	1.805	228.266	.13	.917	.206	1.144	
5.654	11.000	.500	2.000	2.184	275.910	.16	1.110	.250	1.042	
6.168	12.000	.500	2.000	2.599	328.066	.19	1.321	.297	.956	
6.682	13.000	.500	2.000	3.050	384.734	.22	1.550	.349	.883	
7.196	14.000	.500	2.000	3.538	445.914	.26	1.798	.404	.820	
7.710	15.000	.500	2.000	4.061	511.607	.30	2.064	.464	.766	
8.224	16.000	.500	2.000	4.620	581.811	.34	2.348	.528	.718	
8.738	17.000	.500	2.000	5.214	656.527	.38	2.651	.596	.676	
9.252	18.000	.500	2.000	5.843	735.769	.42	2.972	.669	.639	
9.766	19.000	.500	2.000	6.516	819.498	.47	3.312	.745	.606	
10.280	20.000	.500	2.000	7.220	907.752	.52	3.669	.825	.576	

K_s = .500+02 NT/(M/S)2 = .297+01 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.048= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)	
1.028	2.000	.500	2.000	.103	13.447	.01	.053	.012	3.911	
1.542	3.000	.500	2.000	.232	29.853	.02	.119	.027	2.643	
2.056	4.000	.500	2.000	.413	52.715	.03	.211	.048	1.996	
2.570	5.000	.500	2.000	.645	82.027	.05	.330	.074	1.603	
3.084	6.000	.500	2.000	.929	117.786	.07	.476	.107	1.340	
3.598	7.000	.500	2.000	1.265	160.019	.09	.647	.146	1.150	TENTH-
4.112	8.000	.500	2.000	1.654	208.697	.12	.845	.190	1.008	SIZE
4.626	9.000	.500	2.000	2.091	263.830	.15	1.070	.241	.897	
5.140	10.000	.500	2.000	2.582	323.416	.19	1.321	.297	.808	1 Duct
5.654	11.000	.500	2.000	3.124	393.458	.23	1.598	.360	.735	
6.168	12.000	.500	2.000	3.718	467.953	.27	1.902	.428	.674	
6.682	13.000	.500	2.000	4.363	548.904	.32	2.233	.503		
7.196	14.000	.500	2.000	5.059	636.308	.37	2.589	.583	.540	
7.710	15.000	.500	2.000	5.809	730.167	.42	2.972	.669	.507	
8.224	16.000	.500	2.000	6.461	830.481	.48	3.381	.761	.477	
8.738	17.000	.500	2.000	7.461	937.249	.54	3.818	.859	.451	
9.252	18.000	.500	2.000	8.365	1050.472	.61	4.280	.963	.427	
9.766	19.000	.500	2.000	9.320	1170.149	.68	4.769	1.073	.406	
10.327	20.000	.500	2.000	10.327	1296.280	.75	5.284	1.189		

K_s = .125+02 NT/(M/S)2 = .743+00 LB/(KT)2 ***AREA = 1.000 SQ METERS***K1 = 1.012= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)	
1.028	2.000	.500	2.000	.026	3.803	.00	.013	.003	13.600	
1.542	3.000	.500	2.000	.059	8.166	.00	.030	.007	9.500	
2.056	4.000	.500	2.000	.105	14.170	.01	.053	.012	7.299	
2.570	5.000	.500	2.000	.164	21.815	.01	.083	.019	5.927	
3.084	6.000	.500	2.000	.236	31.102	.02	.116	.027	4.988	
3.598	7.000	.500	2.000	.322	42.029	.02	.162	.036	4.307	
4.112	8.000	.500	2.000	.420	54.598	.03	.211	.048	3.789	
4.626	9.000	.500	2.000	.532	68.808	.04	.267	.060	3.382	4 Ducts

K_S =   .556+01 NT/(M/S)2 =   .330+00 LB/(KT)2 •••AREA =   1.000 SQ METERS•••K1 =   1.006= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)

9 DUCTS

≈ K_S =   .312+01 NT/(M/S)2 =   .186+00 LB/(KT)2 •••AREA =   1.000 SQ METERS•••K1 =   1.003= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)

16 DUCTS

K_S =   .500+04 NT/(M/S)2 =   .297+03 LB/(KT)2 •••AREA =   .250 SQ METERS•••K1 =   5.000= RATIO VIN/VS

VS(M/S)	VS(KTS)	B(TESLA)	L(METERS)	CUR(KAMP)	VOLTS	PRESS(PSI)	TH(KNTS)	TH(KLBS)	EFFICIENCY(%)
1.028	2.000	.500	10.000	12.681	318.320	1.84	5.284	1.189	.135
1.542	3.000	.500	10.000	25.733	715.357	4.14	11.889	2.674	.090
2.056	4.000	.500	10.000	50.760	1270.911	7.36	21.136	4.754	.067
2.570	5.000	.500	10.000	79.720	1984.682	11.40	33.026	7.428	.054
3.084	6.000	.500	10.000	114.533	2857.172	16.55	47.555	10.697	.045
4.112	8.000	.500	10.000	155.347	3888.179	22.53	64.728	14.560	.039
4.626	9.000	.500	10.000	202.903	5077.703	29.42	84.543	19.017	.030
5.140	10.000	.500	10.000	256.798	6425.745	37.24	106.999	24.068	.027
5.654	11.000	.500	10.000	317.035	7933.305	45.97	132.098	29.714	.027
		.500	10.000	383.613	9597.382	55.62	159.839	35.953	.025